Name Answer Key **Date** ______________

Calc I H – 1.2-1.3 Practice **Period** ______

Evaluate each limit.

1. $\lim_{x\to 2}\frac{x^2-3x-4}{x-2}$ $\lim_{x\to 2^-} f(x)$ *Be sure to show and use the table below.* $\lim_{x\to 2^+} f(x)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
y	60.9	600.99	6001	error	-5999	-599	-58.9

$\lim_{x\to 2^-}\frac{x^2-3x-4}{x-2}=$ ∞ $\lim_{x\to 2^+}\frac{x^2-3x-4}{x-2}=$ $-\infty$ $\lim_{x\to 2}\frac{x^2-3x-4}{x-2}=$ DNE

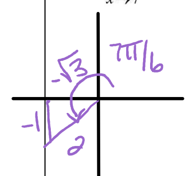
2. $\lim_{x\to 3}\frac{2x-3}{x+5}=\frac{2(3)-3}{3+5}=\boxed{\frac{3}{8}}$

$\lim_{x\to 3}\frac{x^2-3}{x}=\frac{3^2-3}{3}=\frac{6}{3}=\boxed{2}$

3. $\lim_{x\to 1}\sin\frac{\pi x}{2}=\sin\left(\frac{\pi}{2}\right)=\boxed{1}$

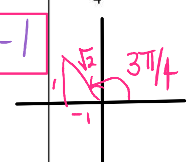
$\lim_{x\to 6}\cos\frac{\pi x}{2}=\cos\left(\frac{6\pi}{2}\right)=\cos(3\pi)=\boxed{-1}$

4. $\lim_{x\to 7}\sec\frac{\pi x}{6}=\sec\frac{7\pi}{6}$



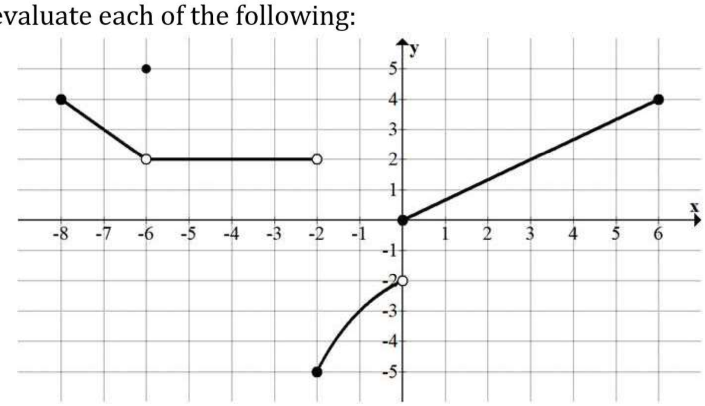
$=\boxed{-\frac{2}{\sqrt{3}}}$

$\lim_{x\to\frac{\pi}{4}}\frac{\tan 3x}{2}=\frac{\tan\frac{3\pi}{4}}{2}$



$=\boxed{\frac{-1}{2}}$

5. Use the graph of $f(x)$ below to evaluate each of the following:



(a) $f(-6)=$ 5

(b) $\lim_{x\to -6} f(x)=$ 2

(c) $f(-2)=$ -5

(d) $\lim_{x\to -2^-} f(x)=$ 2

(e) $\lim_{x\to -2^+} f(x)=$ -5

(f) $\lim_{x\to -2} f(x)=$ DNE

(g) $f(0)=$ 0

(h) $\lim_{x\to 0^-} f(x)=$ -2

(i) $\lim_{x\to 0^+} f(x)=$ 0

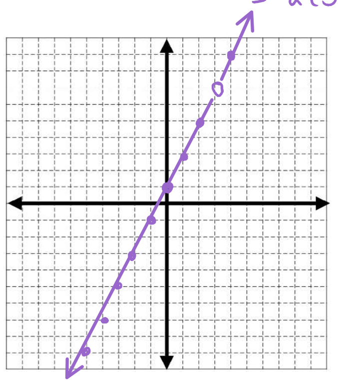
(j) $\lim_{x\to 0} f(x)=$ DNE

Evaluate each limit.

6. $\lim\limits_{x\to3}\dfrac{2x^2-5x-3}{x-3}$ *Show a sketch below.*

$$\lim_{x\to3}\frac{(2x+1)\cancel{(x-3)}}{\cancel{x-3}}=\lim_{x\to3}(2x+1)$$

$$=2(3)+1=\boxed{7}$$

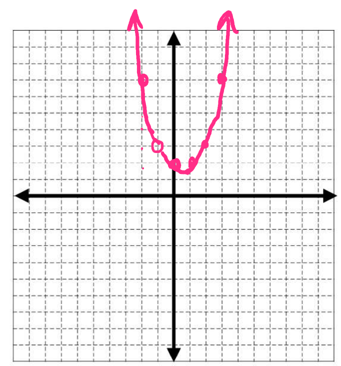


7. $\lim\limits_{x\to-1}\dfrac{x^3+1}{x+1}$ $\left[\text{Hint: } a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$$=\lim_{x\to-1}\frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}=\lim_{x\to-1}(x^2-x+1)$$

$$=(-1)^2-(-1)+1$$

$$=3$$



8. $\lim\limits_{x\to0}\dfrac{\sqrt{3+x}-\sqrt{3}}{x}=\dfrac{\sqrt3-\sqrt3}{0}=\text{"}\dfrac{0}{0}\text{"}$

$$\lim_{x\to0}\frac{(\sqrt{3+x}-\sqrt3)}{x}\cdot\frac{(\sqrt{3+x}+\sqrt3)}{(\sqrt{3+x}+\sqrt3)}$$

$$=\lim_{x\to0}\frac{\cancel{3}+x-\cancel{3}}{x(\sqrt{3+x}+\sqrt3)}$$

$$=\lim_{x\to0}\frac{\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt3)}=\lim_{x\to0}\frac{1}{\sqrt{3+x}+\sqrt3}$$

$$=\frac{1}{\sqrt3+\sqrt3}=\boxed{\frac{1}{2\sqrt3}}$$

9. $\lim\limits_{x\to0}\dfrac{\frac{1}{x+5}-\frac{1}{5}}{x}=\dfrac{\frac15-\frac15}{0}=\text{"}\dfrac{0}{0}\text{"}$

$$\lim_{x\to0}\frac{\left(\frac{1}{x+5}-\frac{1}{5}\right)\cdot5(x+5)}{x\cdot5(x+5)}$$

$$=\lim_{x\to0}\frac{\cancel{5}-(x+\cancel{5})}{5x(x+5)}=\lim_{x\to0}\frac{-\cancel{x}}{5\cancel{x}(x+5)}$$

$$=\lim_{x\to0}\frac{-1}{5(x+5)}=\frac{-1}{5(5)}$$

$$=\boxed{\frac{-1}{25}}$$

10. $\lim\limits_{x\to0}\dfrac{\sin(5x)}{x}=\dfrac{\sin(0)}{0}=\text{"}\dfrac{0}{0}\text{"}$

$$=\lim_{x\to0}\frac{5\cdot\sin(5x)}{5\cdot x}$$

$$=5\lim_{x\to0}\frac{\sin(5x)}{5x}$$

$$=5\cdot1=\boxed{5}$$

11. $\lim\limits_{x\to0}\dfrac{\sin x}{5x}=\dfrac{\sin(0)}{0}=\text{"}\dfrac{0}{0}\text{"}$

$$\lim_{x\to0}\frac{\sin x}{5x}=\frac{1}{5}\lim_{x\to0}\frac{\sin x}{x}$$

$$=\frac15\cdot1=\boxed{\frac15}$$

12. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \frac{\sqrt{4}-2}{3-3} = \text{"}\frac{0}{0}\text{"}$

$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$

$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

13. $\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x} = \frac{\frac{1}{3}-\frac{1}{3}}{0}$ $\text{"}\frac{0}{0}\text{"}$

$\lim_{x\to 0}\frac{\left(\frac{1}{3+x}-\frac{1}{3}\right)\cdot 3(3+x)}{x\cdot 3(3+x)}$

$= \lim_{x\to 0}\frac{\cancel{3}-(\cancel{3}+x)}{3x(3+x)} = \lim_{x\to 0}\frac{-\cancel{x}}{3\cancel{x}(3+x)}$

$= \lim_{x\to 0}\frac{-1}{3(3+x)} = \frac{-1}{3(3)} = \boxed{-\frac{1}{9}}$

14. $\lim_{x\to 2}\frac{2-x}{x^2-4} = \frac{2-2}{2^2-4} = \text{"}\frac{0}{0}\text{"}$

$\lim_{x\to 2}\frac{\cancel{2-x}\;-1}{(x+2)\cancel{(x-2)}} = \lim_{x\to 2}\frac{-1}{x+2}$

$= \frac{-1}{2+2} = \boxed{-\frac{1}{4}}$

15. $\lim_{x\to 0}\frac{\sin(2x)}{5x}$ $\left[\text{Hint: } \lim_{x\to 0}\frac{\sin(2x)}{5x} = \frac{1}{5}\lim_{x\to 0}\frac{\sin(2x)}{x}\right]$ $\text{"}\frac{0}{0}\text{"}$

$= \frac{1}{5}\lim_{x\to 0}\frac{2\cdot\sin(2x)}{2\cdot x}$

$= \frac{2}{5}\lim_{x\to 0}\frac{\sin(2x)}{2x}$

$= \frac{2}{5}\cdot 1 = \boxed{\frac{2}{5}}$

$\text{"}\frac{0}{0}\text{"}$

16. $\lim_{x\to 2}\frac{x^3-8}{x-2}$ $\left[\text{Hint: } a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$= \lim_{x\to 2}(x^2+2x+4) = 2^2+2(2)+4$

$= \boxed{12}$

17. $\lim_{x\to\pi/4}\frac{4x}{\tan x} = \frac{4(\pi/4)}{\tan \pi/4}$

$= \frac{\pi}{1} = \boxed{\pi}$

18. $\lim_{x\to 0}\frac{1-\cos x}{10x} = \frac{1-1}{0} = \text{"}\frac{0}{0}\text{"}$

$\frac{1}{10}\lim_{x\to 0}\frac{1-\cos x}{x}$

$= \frac{1}{10}(0) = \boxed{0}$

19. $\lim_{x\to 4} 3|1-x|$

$= 3|1-4|$

$= 3|-3|$

$= 3(3) = \boxed{9}$

20. Evaluate each without drawing the graph given: $f(x)=\begin{cases}5x-2, & x\le -2\\ x^3, & -2<x<0\\ -x, & 0<x<1\\ 2, & x=1\\ 2x^2-3, & x>1\end{cases}$

a. $f(-2)=$ $\underline{-12}$ $5(-2)-2$

b. $f(0)=$ $\underline{\text{und}}$

c. $f(1)=$ $\underline{2}$

$\lim_{x\to -2^-} f(x)=\underline{-12}$

$\lim_{x\to 0^-} f(x)=\underline{0}$ 0^3

$\lim_{x\to 1^-} f(x)=\underline{-1}$ -1

$\lim_{x\to -2^+} f(x)=\underline{-8}$ $(-2)^3$

$\lim_{x\to 0^+} f(x)=\underline{0}$ -0

$\lim_{x\to 1^+} f(x)=\underline{-1}$ $2(1)^2-3$

$\lim_{x\to -2} f(x)=\underline{\text{DNE}}$

$\lim_{x\to 0} f(x)=\underline{0}$

$\lim_{x\to 1} f(x)=\underline{-1}$

21. Evaluate each without drawing the graph given: $g(x)=\begin{cases}-8, & x\le -6\\ 3x+10, & -6<x<-2\\ -5, & x=-2\\ x^2, & -2<x\le 3\\ -2x+9, & x>3\end{cases}$

a. $g(-6)=\underline{-8}$

b. $g(-2)=\underline{-5}$

c. $g(3)=\underline{9}$ 3^2

$\lim_{x\to -6^-} g(x)=\underline{-8}$

$\lim_{x\to -2^-} g(x)=\underline{4}$ $3(-2)+10$

$\lim_{x\to 3^-} g(x)=\underline{9}$

$\lim_{x\to -6^+} g(x)=\underline{-8}$ $3(-6)+10$

$\lim_{x\to -2^+} g(x)=\underline{4}$ $(-2)^2$

$\lim_{x\to 3^+} g(x)=\underline{3}$ $-2(3)+9$

$\lim_{x\to -6} g(x)=\underline{-8}$

$\lim_{x\to -2} g(x)=\underline{4}$

$\lim_{x\to 3} g(x)=\underline{\text{DNE}}$

22. *Challenge* Evaluate each of the following:

a. $\lim_{x\to 0}\frac{\sin 5x}{\sin 2x} = \lim_{x\to 0}\frac{5\sin 5x}{5x}\cdot\frac{2x}{2\sin 2x}$

$= \frac{5}{2}\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\cdot\frac{2x}{2\sin 2x}\right)$

$= \frac{5}{2}\cdot 1\cdot 1 = \boxed{\frac{5}{2}}$

b. $\lim_{x\to 0}\frac{1-\cos x}{\sin 4x} = \lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{4x}{4\sin 4x}$

$= \frac{1}{4}\lim_{x\to 0}\left(\frac{1-\cos x}{x}\cdot\frac{4x}{\sin 4x}\right)$

$= \frac{1}{4}\cdot 0\cdot 1 = \boxed{0}$