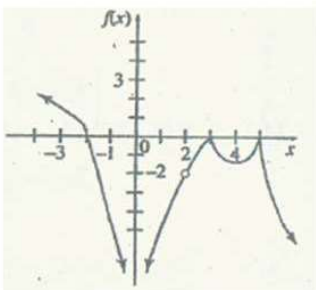
Name Answer Key   Date________

Calc I H – 1.3-1.4 Review   **Period______**

For 1-9, identify ALL values of *x* where the function is discontinuous, the type of each discontinuity and the conditions of continuity that are not satisfied.

1.)

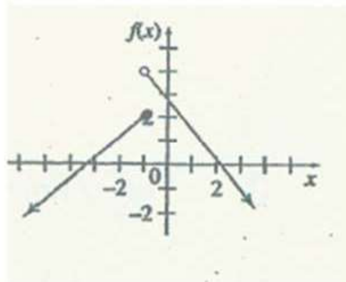


$x=0$, Non-Remov Infinite disc

$\lim_{x\to 0^-} f(x)=-\infty$, $\lim_{x\to 0^+} f(x)=-\infty$, $f(0)$ und

$x=2$, Removable hole, $f(2)$ und, $\lim_{x\to 2} f(x)=-2$

2.)



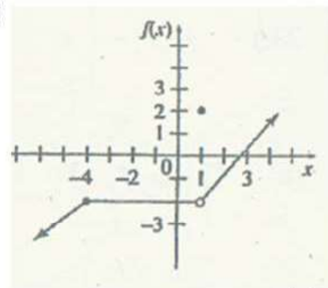
$x=-1$

Non-Remov Jump disc

$\lim_{x\to -1^-} f(x)=2 \neq \lim_{x\to -1^+} f(x)=4$, $f(-1)=2$

$\lim_{x\to -1} f(x)$ DNE,

3.)



$x=1$ Remov Hole disc.

$f(1)=2 \neq \lim_{x\to 1} f(x)=-2$

4.) $f(x)=\dfrac{5+x}{x(x-2)}$

$x=0$ & $x=2$, Non Remov Inf Disc

$\lim_{x\to 0^-} f(x)=\infty$ $\lim_{x\to 0^+} f(x)=-\infty$, $f(0)$ und

$\lim_{x\to 2^-} f(x)=-\infty$ $\lim_{x\to 2^+} f(x)=\infty$, $f(2)$ und

5.) $f(x)=\dfrac{x^2-4}{x-2}=\dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$, $x\neq 2$

$x=2$ Removable Hole disc

$f(2)$ und, $\lim_{x\to 2} f(x)=4$

6. $f(x)=x^2-4x+11$

Continuous on $(-\infty,\infty)$

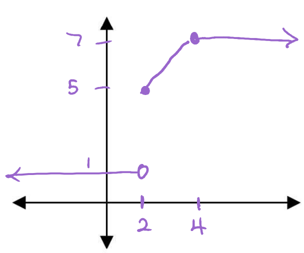
7. $f(x)=\dfrac{|x+2|}{x+2}$

$x=-2$, Non Remov, Jump disc

$\lim_{x\to -2^-} f(x)=-1 \neq \lim_{x\to -2^+} f(x)=1$

$\lim_{x\to -2} f(x)$ DNE

8. $f(x)=\begin{cases} 1 & \text{if } x<2 \\ x+3 & \text{if } 2\le x\le 4 \\ 7 & \text{if } x>4 \end{cases}$



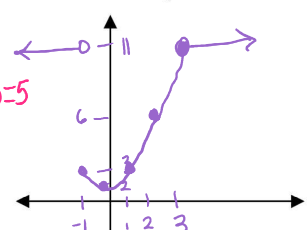
$x=2$, Non Remov Jump disc

$\lim_{x\to 2^-} f(x)=1 \neq \lim_{x\to 2^+} f(x)=5$

$\lim_{x\to 2} f(x)$ DNE

$f(2)=5$

9. $f(x)=\begin{cases} 11 & \text{if } x<-1 \\ x^2+2 & \text{if } -1\le x\le 3 \\ 11 & \text{if } x>3 \end{cases}$



$x=-1$ Non Remov jump disc

$\lim_{x\to -1^-} f(x)=11 \neq \lim_{x\to -1^+} f(x)=3$

$\lim_{x\to -1} f(x)$ DNE

$f(-1)=3$

$4x^3-3\left(\frac{37}{6}\right)x+2 = 4x^3-\frac{37}{2}x+2$

10. **Determine the value of *c* so that $f(x)$ is continuous on the entire real line:**

a) $f(x)=\begin{cases} x-2, & x\le 5 \\ cx-3, & x>5 \end{cases}$

Check: $f(5)=3$

$\lim_{x\to5^-} f(x)=3=\lim_{x\to5^+} f(x)=\frac{6}{5}(5)-3$

$\lim_{x\to5} f(x)=3$

$f(5)=\lim_{x\to5} f(x)$ ✓

$\lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x)$

$5-2 = 5c-3$

$3 = 5c-3$

$6=5c$

$\boxed{c=\frac{6}{5}}$

b) $f(x)=\begin{cases} x^2-7, & x\le 2 \\ 4x^3-3cx+2, & x>2 \end{cases}$

Check: $f(2)=-3$

$\lim_{x\to2^-} f(x)=-3$

$\lim_{x\to2^+} f(x)=4(2)^3-\frac{37}{2}(2)+2$

$=-3$

$\lim_{x\to2} f(x)=-3$

$f(2)=\lim_{x\to2} f(x)$ ✓

$\lim_{x\to2^-} f(x) = \lim_{x\to2^+} f(x)$

$2^2-7 = 4(2)^3-3(2)c+2$

$-3 = 32-6c+2$

$-3 = 34-6c$

$-37=-6c$

$c=37/6$

For 11-12, determine the indicated limits using any method.

11. $f(x)=\begin{cases} -5, & x<-3 \\ x, & -3\le x\le 3 \\ 5, & x>3 \end{cases}$

(a) $\lim_{x\to-3^-} f(x)=$ -5

(b) $\lim_{x\to-3^+} f(x)=$ -3

(c) $\lim_{x\to-3} f(x)=$ DNE

(d) $\lim_{x\to3^-} f(x)=$ 3

(e) $\lim_{x\to3^+} f(x)=$ 5

(f) $\lim_{x\to3} f(x)=$ DNE

12. $f(x)=\begin{cases} x, & x<1 \\ x^2, & 1\le x<2 \\ 5, & x\ge 2 \end{cases}$

(a) $\lim_{x\to1^-} f(x)=$ 1

(b) $\lim_{x\to1^+} f(x)=$ $1^2=1$

(c) $\lim_{x\to1} f(x)=$ 1

(d) $\lim_{x\to2^-} f(x)=$ $2^2=4$

(e) $\lim_{x\to2^+} f(x)=$ 5

(f) $\lim_{x\to2} f(x)=$ DNE

13. Use the Intermediate Value Theorem to justify that the function $f(x)=x^4-2x^2+3x$ has a zero in the interval $[-2,-1]$.

$f(x)$ is continuous on $[-2,-1]$ since $f(x)$ is a polynomial function.

$f(-2)=(-2)^4-2(-2)^2+3(-2)=2>0$

$f(-1)=(-1)^4-2(-1)^2+3(-1)=-4<0$

Since $f(x)$ is cont on $[-2,-1]$ and $f(-1)<0<f(-2)$, by the IVT there MUST exist $x=c$ on $[-2,-1]$ such that $f(c)=0$.

14. Use the Intermediate Value Theorem to justify that the function $g(x) = \sin(x)$ has a $g(c) = \frac{3}{4}$ in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

$g(x) = \sin x$ is continuous everywhere.

$g(\pi/2) = \sin(\pi/2) = 1 > 3/4$

$g(3\pi/2) = \sin(3\pi/2) = -1 < 3/4$

Since $g(x)$ is cont on $[\pi/2, 3\pi/2]$ and $g(3\pi/2) < 3/4 < g(\pi/2)$, there must exist $x = c$ on $[\pi/2, 3\pi/2]$ such that $g(c) = 3/4$.

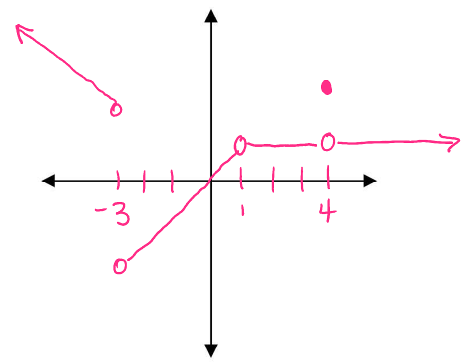
14. Sketch a function that has each of the following characteristics:

Sample sketches ONLY!!

a. Jump Discontinuity @ $x = -3$ and $f(-3) = und$

Removable Discontinuity @ $x = 1$ because $f(1) = und$ (hole)

Removable Discontinuity @ $x = 4$ because $\lim_{x \to 4} f(x) \neq f(4)$ hole)



b. Discontinuity @ $x = -2$ but $\lim_{x \to -2} f(x) = 5$ hole!

Discontinuity @ $x = 1$ because $\lim_{x \to 1^-} f(x) = \infty$ infinite

$\lim_{x \to 1^+} f(x) = -\infty$ (VA)

Discontinuity @ $x = 5$ but $f(5)$ is defined and

$\lim_{x \to 5^+} f(x) = 3$

$\lim_{x \to 5^-} f(x) = -1$ jump