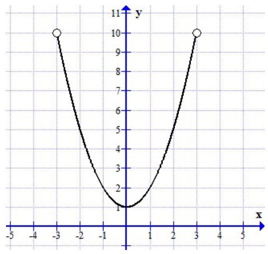
Do Now:
Using your notes from last night, discuss the continuity of the following functions on the given interval. You may use a calculator, but be sure you could graph each function by hand.

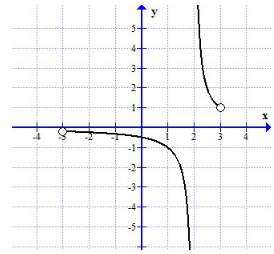
a) $f(x)=x^2+1$ on $(-3,3)$

Continuous on $(-3,3)$



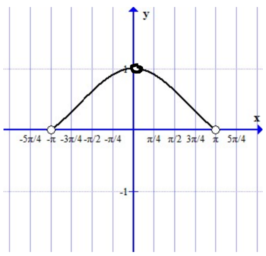
b) $g(x)=\dfrac{1}{x-2}$ on $(-3,3)$

Discontinuity at $x=2$
- non-removable (Infinite)
- $\lim\limits_{x\to 2} g(x) = \text{DNE}$



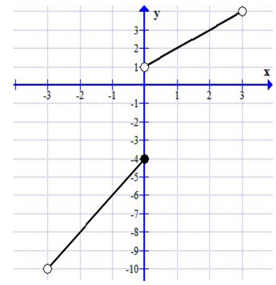
c) $h(x)=\dfrac{\sin(x)}{x}$ on $(-\pi,\pi)$

Discontinuity at $x=0$:
- removable (hole)
- $\lim\limits_{x\to 0} h(x)=1$
- $f(0)$ is undefined



d) $p(x)=\begin{cases}2x-4, & x\le 0\\ x+1, & x>0\end{cases}$ on $(-3,3)$

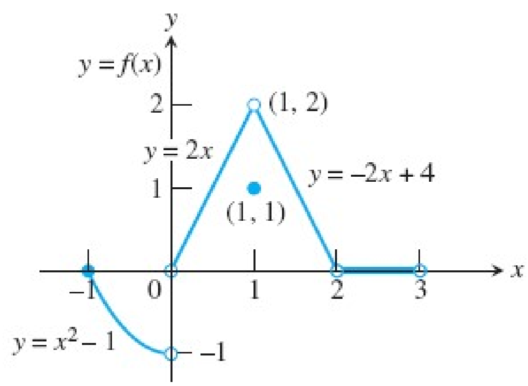
Discontinuity at $x=0$:
- non-removable (Jump)
- $\lim\limits_{x\to 0} p(x) = \text{DNE}$



Sample Problems:

1. Discuss the continuity of the given function *f(x)* on the interval [-1, 3). Justify all points of discontinuity using the definition.

$$f(x)=\begin{cases}x^2-1, & -1\le x<0\\ 2x, & 0<x<1\\ 1, & x=1\\ -2x+4, & 1<x<2\\ 0, & 2<x<3\end{cases}$$



$x=0$: non-removable (Jump) → $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

$x=1$: removable (hole) → $\lim\limits_{x\to 1} f(x) = 2 \neq 1 = f(1)$

$x=2$: removable (hole) → $\lim\limits_{x\to 2} f(x)=0$ but $f(2)$ is undefined

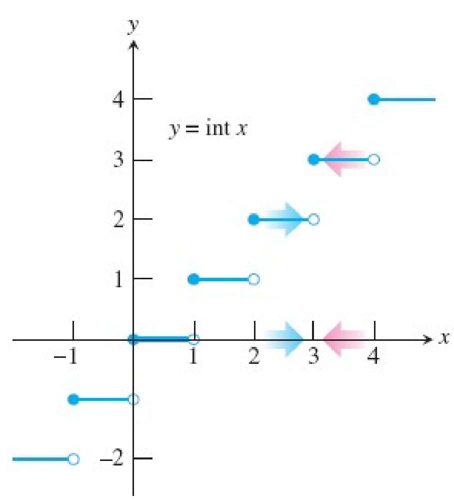
2. Discuss the continuity for the Greatest Integer function.

Discontinuous at every integer:

not continuous on its domain

For any integer n,

$\left.\begin{array}{l}\lim\limits_{x\to n^-} f(x)=n-1\\ \lim\limits_{x\to n^+} f(x)=n\end{array}\right\}\ \lim\limits_{x\to n} f(x)$ DNE

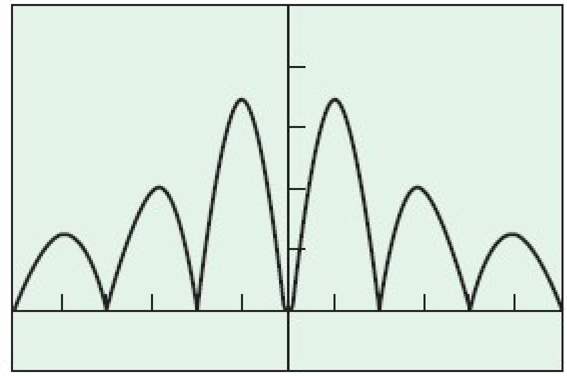


Continuous for all non-integers:

For any non-integer, real number d,

$\left.\begin{array}{l}\lim\limits_{x\to d} [\![x]\!] = [\![d]\!]\\ f(d)=[\![d]\!]\end{array}\right\}\ \lim\limits_{x\to d} f(x)=f(d)$

3. Show that $f(x)=\left|\dfrac{x\sin x}{x^2+2}\right|$ is continuous.



Let $g(x)=|x|$ and $h(x)=\dfrac{x\sin x}{x^2+2}$. Then, $f(x)=g(h(x))$.

$g(x)$ is continuous on $(-\infty,\infty)$

$h(x)$ is continuous since it's the product, quotient and sum of the continuous functions x, $\sin x$, x^2 and 2 and $x^2+2\neq 0$.

The composition of continuous functions is also continuous, so $f(x)$ is continuous

4. Which of the following points of discontinuity of $f(x)=\dfrac{\cancel{x}\cancel{(x-1)}(x-2)^2\cancel{(x+1)^2}(x-3)^2}{\cancel{x}\cancel{(x-1)}(x-2)\cancel{(x+1)^2}(x-3)^3}$ is not removable? Use the definition of continuity to justify the type of discontinuity.

$f(x)=\dfrac{(x-2)}{(x-3)}$

a. $x=-1$ hole

b. $x=0$ hole

c. $x=1$ hole

d. $x=2$ hole

e. $x=3$ V.A.

5. Which of the following statements about the function $f(x)=\begin{cases}2x, & 0<x<1\\ 1, & x=1\\ -x+3, & 1<x<2\end{cases}$ is not true? Use the definition of continuity to identify and classify any discontinuities.

D: $(0,2)$

$x=1$ removable (hole) discontinuity

$\lim\limits_{x\to 1} f(x)=2\neq 1=f(1)$

a. $f(1)$ does not exist. $f(1)=1$

b. $\lim\limits_{x\to 0^+} f(x)$ exists. $\lim\limits_{x\to 0^+} f(x)=2(0)=0$ True!

c. $\lim\limits_{x\to 2^-} f(x)$ exists. $\lim\limits_{x\to 2^-} f(x)=-2+3=1$ True!

d. $\lim\limits_{x\to 1} f(x)$ exists. $\lim\limits_{x\to 1^-} f(x)=2$ $\lim\limits_{x\to 1^+} f(x)=2$ True!

e. $\lim\limits_{x\to 1} f(x)\neq f(1)$ True!

6. On which of the following intervals is $f(x)=\dfrac{1}{\sqrt{x}}$ not continuous? Why?

a. $(0,\infty)$

b. $[0,\infty)$ $f(0)$ is undefined

c. $(0,2)$

d. $(1,2)$

e. $[1,\infty)$