Name Answer Key Date______________

Calc I H – 1.4 day 3 – Continuity & IVT Group Work **Period______**

1. Use the IVT to prove that $g(x) = x^2 - 3x + 1$ has a 0 in the interval [-1,2].

g(x) is continuous on [-1,2] since it is a polynomial function.

$g(-1) = (-1)^2 - 3(-1) + 1 = 6 > 0$
$g(2) = (2)^2 - 3(2) + 1 = -1 < 0$

Since g(x) is cont on [-1,2] and $g(2) < 0 < g(-1)$, by the IVT there exists x=c such that g(c)=0.

2. Use the IVT to prove that $h(x) = \frac{x-3}{x^2-9}$ has $h(c) = \frac{1}{2}$ in the interval [-2,2].

h(x) is continuous everywhere except when $x = \pm 3$, ∴ h(x) is cont on [-2,2].

$h(-2) = \frac{-2-3}{(-2)^2-9} = \frac{-5}{-5} = 1$

$h(2) = \frac{2-3}{2^2-9} = \frac{-1}{-5} = \frac{1}{5}$

Since h(x) is cont on [-2,2] and $h(2) < \frac{1}{2} < h(-2)$ by the IVT there exists x=c such that $h(c) = \frac{1}{2}$.

3. Find a so that each function is continuous on $(-\infty, \infty)$. Use the definition of continuity to support your answer.

a. $h(x) = \begin{cases} ax+4, & x<3 \\ 7-2x, & x \ge 3 \end{cases}$

$\lim_{x\to 3^-} h(x) = \lim_{x\to 3^+} h(x)$

$3a + 4 = 7 - 2(3)$

$3a + 4 = 1$

$3a = -3$

$\boxed{a = -1}$

when $a = -1$, $h(3) = -(3) + 4 = 1$

$\lim_{x\to 3^-} h(x) = \lim_{x\to 3^+} h(x) = 1$

$\therefore h(3) = \lim_{x\to 3} h(x)$ ✓

b. $g(x) = \begin{cases} \frac{2}{3}x - 5, & x \le -1 \\ ax+2, & x > -1 \end{cases}$

$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^+} g(x)$

$-\frac{2}{3} - 5 = -a + 2$

$-\frac{17}{3} = -a + \frac{6}{3}$

$-a = -\frac{23}{3}$

$a = \frac{23}{3}$

when $a = \frac{23}{3}$, $g(-1) = -\frac{2}{3} - 5 = -\frac{17}{3}$

$\lim_{x\to -1^-} g(x) = -\frac{17}{3} = \lim_{x\to -1^+} g(x) = -\frac{23}{3} + 2$

$\therefore g(-1) = \lim_{x\to -1} g(x)$ ✓

4. Explain why you cannot use the Intermediate Value Theorem to prove that $p(x) = \frac{1}{x-2}$ has a 0 in the interval [1,4].

Since p(x) is undefined at x=2, p(x) is NOT continuous on [1,4] so IVT does not apply.

5. Find a and b so that the function is continuous on the entire real number line.

$$p(x)=\begin{cases}3, & x<-2\\ ax+b, & -2\le x\le 2 \Rightarrow \frac{1}{2}x+4\\ 5, & x>2\end{cases}$$

$\lim_{x\to -2^-} p(x) = \lim_{x\to -2^+} p(x)$

$3 = -2a+b$

$\lim_{x\to 2^-} p(x) = \lim_{x\to 2^+} p(x)$

$2a+b=5$

$3=-\cancel{2a}+b$
$+\ 5=\cancel{2a}+b$

$8=2b$

$\boxed{4=b}$

Then $3=-2a+4$

$-1=-2a$

$\boxed{\frac{1}{2}=a}$

Check:

$p(-2)=\frac{1}{2}(-2)+4=3$

$\lim_{x\to -2^-} p(x)=3=\lim_{x\to 2^+} p(x)$

So $p(-2)=\lim_{x\to -2} p(x)$ ✓

$p(2)=\frac{1}{2}(2)+4=5$

$\lim_{x\to 2^-} p(x)=5=\lim_{x\to 2^+} p(x)$

So $p(2)=\lim_{x\to 2} p(x)$ ✓

6. For each function below identify any discontinuities and give the type and reason for each.

a. $f(x)=\frac{1}{x-2}$ Nonremovable, infinite disc at $x=2$ $\lim_{x\to 2^-} f(x)=-\infty$ $\lim_{x\to 2^+} f(x)=\infty$ $\lim_{x\to 2} f(x)$ DNE $f(2)$ und	b. $g(x)=\frac{x^2-x+6}{x^2+x-12}$ $g(x)=\frac{(\cancel{x-3})(x+2)}{(x+4)(\cancel{x-3})}=\frac{x+2}{x+4}$ Removable hole disc at $x=3$ $f(3)$ und, $\lim_{x\to 3} f(x)=\frac{5}{7}$ Nonremovable infinite disc at $x=-4$ $\lim_{x\to -4^-} f(x)=\infty$ $\lim_{x\to -4^+} f(x)=-\infty$ $\lim_{x\to -4} f(x)$ DNE $f(-4)$ und	c. $h(x)=\begin{cases}3x+4, & x\le 1\\ x^2-5, & x>1\end{cases}$ $\lim_{x\to 1^-} h(x)=7$, $\lim_{x\to 1^+} h(x)=-4$ $\lim_{x\to 1} h(x)$ DNE $\Rightarrow$ Nonremovable Jump disc at $x=1$ $h(1)=7$
d. $p(x)=\frac{3}{x+4}$ Nonremovable Infinite disc at $x=-4$ $\lim_{x\to -4^-} p(x)=-\infty$ $\lim_{x\to -4^+} p(x)=\infty$ $\lim_{x\to -4} p(x)$ DNE $p(-4)$ und	e. $q(x)=\frac{2x^2-7x-4}{x^2+2x-24}$ $q(x)=\frac{(2x+1)(\cancel{x-4})}{(x+6)(\cancel{x-4})}=\frac{2x+1}{x+6}$ Removable hole disc at $x=4$ $q(4)$ und Nonremovable infinite disc at $x=-6$ $\lim_{x\to -6^-} q(x)=\infty$ $\lim_{x\to -6^+} q(x)=-\infty$ $\lim_{x\to -6} q(x)$ DNE $q(-6)$ und	f. $r(x)=\begin{cases}-2x, & x\le -3\\ -x+3, & -3<x<2\\ x^3, & x\ge 2\end{cases}$ $\lim_{x\to -3^-} r(x)=6=\lim_{x\to -3^+} r(x)$ $r(-3)=6=\lim_{x\to -3} r(x)$ Continuous at $x=-3$ Nonremovable, jump disc at $x=2$ $\lim_{x\to 2^-} r(x)=-1\neq \lim_{x\to 2^+} r(x)=8$ $r(2)=8$