Do Now: IN YOUR NOTEBOOKS!

1. Let the function *f(x)* be defined by: $f(x)=\dfrac{x^2-x-6}{x^2-4}$. Analyze the function completely and then sketch the function. Using what we know about limits, create four limits involving infinity and *f(x)*.

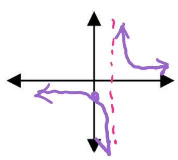
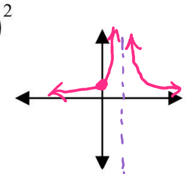
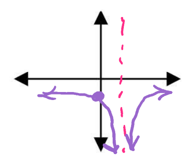
2. Create a definition for both a horizontal and vertical asymptote that utilizes limits.

Sample Problems:

1. Evaluate the following limits:

   a) $\lim\limits_{x\to 0^-}\dfrac{1}{x} = -\infty$ b) $\lim\limits_{x\to 0^+}\dfrac{1}{x} = \infty$ c) $\lim\limits_{x\to \infty}\dfrac{1}{x} = 0$ d) $\lim\limits_{x\to -\infty}\dfrac{1}{x} = 0$

2. For each of the following rational functions below, determine the limit as x approaches $\infty, -\infty$, and 1 from the left and from the right.

a. $f(x)=\dfrac{1}{x-1}$	b. $g(x)=\dfrac{1}{(x-1)^2}$	c. $h(x)=\dfrac{-1}{(x-1)^2}$
		
$\lim\limits_{x\to\infty} f(x) = 0$	$\lim\limits_{x\to\infty} g(x) = 0$	$\lim\limits_{x\to\infty} h(x) = 0$
$\lim\limits_{x\to-\infty} f(x) = 0$	$\lim\limits_{x\to-\infty} g(x) = 0$	$\lim\limits_{x\to-\infty} h(x) = 0$
$\lim\limits_{x\to 1^-} f(x) = -\infty$	$\lim\limits_{x\to 1^-} g(x) = \infty$	$\lim\limits_{x\to 1^-} h(x) = -\infty$
$\lim\limits_{x\to 1^+} f(x) = \infty$	$\lim\limits_{x\to 1^+} g(x) = \infty$	$\lim\limits_{x\to 1^+} h(x) = -\infty$

3. Use calculus to identify the horizontal asymptotes of the following functions:

   $f(x)=\dfrac{x}{\sqrt{x^2+1}}$

   $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{x}{|x|} = 1$

   $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\dfrac{x}{|x|} = -1$

THEOREM 1.15 Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x\to c} f(x) = \infty \quad \text{and} \quad \lim_{x\to c} g(x) = L.$$

1. Sum or difference: $\lim\limits_{x\to c}[f(x) \pm g(x)] = \infty$

2. Product: $\lim\limits_{x\to c}[f(x)g(x)] = \infty,\quad L > 0$

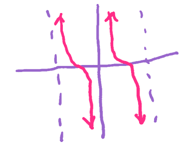
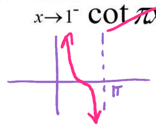
$\lim\limits_{x\to c}[f(x)g(x)] = -\infty,\quad L < 0$

3. Quotient: $\lim\limits_{x\to c}\dfrac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

Practice:

Evaluate the following limits:

1. $\lim\limits_{x\to0}\left(1+\frac{1}{x^2}\right) = \lim\limits_{x\to0} 1 + \lim\limits_{x\to0}\frac{1}{x^2} = 1+\infty = \boxed{\infty}$	2. $\lim\limits_{x\to0^+} 3\cot x = 3\lim\limits_{x\to0^+}\cot x = 3(\infty) = \boxed{\infty}$ 
3. $\lim\limits_{x\to1^-}\frac{x^2+1 \to 2}{\cot \pi x \to \infty} = \boxed{0}$ 	4. $\lim\limits_{x\to\infty}\frac{\sin x}{x}$ $-1\le\sin x\le1$ $-\frac{1}{x}\le\frac{\sin x}{x}\le\frac{1}{x}$ $\lim\limits_{x\to\infty}\left(-\frac{1}{x}\right)=0=\lim\limits_{x\to\infty}\left(\frac{1}{x}\right)$ By the squeeze thm, $\lim\limits_{x\to\infty}\frac{\sin x}{x}=\boxed{0}$
5. $\lim\limits_{x\to\infty}\frac{5x+\sin x}{x} = \lim\limits_{x\to\infty} 5 + \lim\limits_{x\to\infty}\frac{\sin x}{x} = 5+0 = \boxed{5}$	6. $\lim\limits_{x\to-\infty}\frac{1-\cos x}{x^2}$ $-1\le\cos x\le1$ $\frac{2}{x^2}\ge\frac{1-\cos x}{x^2}\ge0$ $\lim\limits_{x\to-\infty}\left(\frac{2}{x^2}\right)=0=\lim\limits_{x\to-\infty}0$ By the squeeze thm, $\lim\limits_{x\to-\infty}\frac{1-\cos x}{x^2}=\boxed{0}$
7. $\lim\limits_{x\to\infty}\frac{2x^5+x^4-x^2+1}{3x^2} = \boxed{\infty}$ $\lim\limits_{x\to\infty}\frac{2x^3+x^2-1+\cancel{\frac{1}{x^2}}\to0}{3}$	8. $\lim\limits_{x\to\infty}\frac{2x^5+x^4-x^2+1}{3x^2-5x+7} = \boxed{\infty}$ $\lim\limits_{x\to\infty}\frac{2x^3-x^2-1+\cancel{\frac{1}{x^2}}\to0}{3-\cancel{\frac{5}{x}}\to0+\cancel{\frac{7}{x^2}}\to0}$ or "$\frac{2x^5}{3x^2}$"
9. $\lim\limits_{x\to\infty}\frac{x+e^{-x}}{x} = \lim\limits_{x\to\infty}\frac{x+\cancel{e^{-x}}\to0}{x} = \boxed{1}$	

10. Sketch a function given the following criteria in your notebook and state the domain and range.

I. $f(5)=-1$ — $(5,-1)$

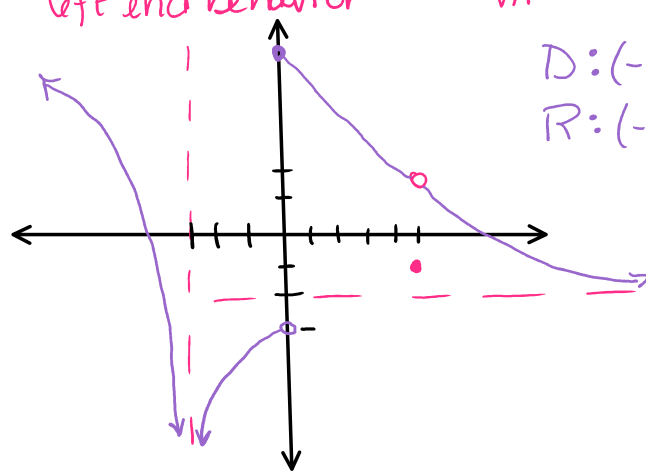
II. $\lim\limits_{x\to5} f(x)=2$ — hole at $(5,2)$

III. $\lim\limits_{x\to0} f(x)$ Does Not Exist — jump at $x=0$

IV. $\lim\limits_{x\to\infty} f(x)=-2$ — HA $y=-2$

V. $\lim\limits_{x\to-\infty} f(x)=\infty$ — left end behavior

VI. $x=-3$ is a vertical asymptote — VA



$D: (-\infty,-3)\cup(-3,\infty)$

$R: (-\infty,\infty)$

NOTE:

Sample answer only!

Graphs, domain & range will vary!

a. $\lim_{x\to 5}\frac{x^2-4}{x^2-25}=$ DNE

$\lim_{x\to 5^-}\frac{(x+2)(x-2)}{(x+5)(x-5)}=-\infty$

$\lim_{x\to 5^+}\frac{(x+2)(x-2)}{(x+5)(x-5)}=\infty$

b. $\lim_{x\to -5}\frac{x^2-4}{x^2-25}=$ DNE

$\lim_{x\to -5^-}\frac{(x+2)(x-2)}{(x+5)(x-5)}=\infty$

$\lim_{x\to -5^+}\frac{(x+2)(x-2)}{(x+5)(x-5)}=-\infty$

c. $\lim_{x\to \infty}\frac{x^2-4}{x^2-25}=1$

$\lim_{x\to\infty}\frac{1-\frac{4}{x^2}\nearrow 0}{1-\frac{25}{x^2}\nearrow 0}$

$y=1$ HA

d. $\lim_{x\to -\infty}\frac{x^2-4}{x^2-25}=1$

OR "$\frac{x^2}{x^2}$"

$y=1$ HA

e. $\lim_{x\to \infty}\frac{4x+2}{x^3}=0$

$\lim_{x\to\infty}\frac{\frac{4}{x^2}\nearrow 0+\frac{2}{x^3}\nearrow 0}{1}$

f. $\lim_{x\to -\infty}\frac{4x+2}{x^3}=0$

OR → "$\frac{4}{x^2}$"

g. $\lim_{x\to \infty}\frac{4x^2-x-5}{x}=\infty$

$\lim_{x\to\infty}\left(4x-1-\frac{5}{x}\nearrow 0\right)$

h. $\lim_{x\to 0}\frac{4x^2-x-5}{x}=$ DNE

$\lim_{x\to 0^-}\left(4x-1-\frac{5}{x}\right)=\infty$

$\lim_{x\to 0^+}\left(4x-1-\frac{5}{x}\right)=-\infty$

$y=0$ HA

i. $\lim_{x\to 1^-}\frac{4x^2-x-5}{x}=\frac{4-1-5}{1}$

$=-2$

Direct Sub!

12. Explain how horizontal asymptotes can help us easily determine limits as $x\to\pm\infty$ for rational functions.

The HA determines end behavior of the function which is the limit as x approaches ∞ (Right end behavior) or $-\infty$ (left end behavior).