Name Answer Key　　　　　　　　　　Date________

Calc I H – 2.1-2.4 Review 1　　　　　　　　　　Period______

1. Find the derivative of $f(x) = 4x^2 - 5x + 1$ by using the <u>limit definition</u> of the derivative.

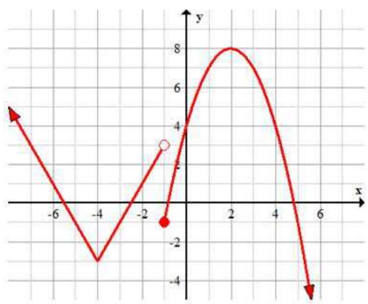
$$f'(x) = \lim_{h\to 0} \frac{4(x+h)^2 - 5(x+h) + 1 - (4x^2 - 5x + 1)}{h} = \lim_{h\to 0} \frac{4(x^2+2xh+h^2) - 5x - 5h + 1 - 4x^2 + 5x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{5x} - 5h + \cancel{1} - \cancel{4x^2} + \cancel{5x} - \cancel{1}}{h} = \lim_{h\to 0} \frac{\cancel{h}(8x+4h-5)}{\cancel{h}}$$

$$= \lim_{h\to 0} (8x+4h-5) = \boxed{8x-5}$$

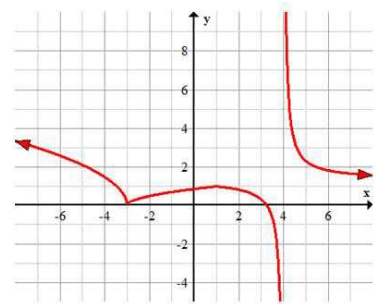
For #2 and #3, describe the *x*-values at which *f* is differentiable.

2.



$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

3.



$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

For #4- #7, find the derivative of the function.

4. $f(x) = \cos(2-4x)^5$

cos( )　( )^5　2-4x

$f'(x) = -\sin(2-4x)^5 \cdot 5(2-4x)^4(-4)$

$\boxed{f'(x) = 20(2-4x)^4 \sin(2-4x)^5}$

NO GCF!

5. $g(x) = \cos^5(2-4x) = (\cos(2-4x))^5$

( )^5　cos( )　2-4x

$g'(x) = 5(\cos(2-4x))^4(-\sin(2-4x))(-4)$

$\boxed{g'(x) = 20\cos^4(2-4x)\sin(2-4x)}$

NO GCF!

6. $j(x) = \dfrac{5x^3}{(2-4x)^5}$

$$j'(x) = \frac{15x^2(2-4x)^5 - \overbrace{5(2-4x)^4(-4)(5x^3)}^{+100x^3(2-4x)^4}}{((2-4x)^5)^2}$$

GCF $5x^2(2-4x)^4$

$6 - 12x + 20x$

$$= \frac{5x^2\cancel{(2-4x)^4}(3(2-4x)+20x)}{(2-4x)^{\cancel{10}\,6}}$$

$$= \boxed{\frac{5x^2(8x+6)}{(2-4x)^6}}$$

7. $j(x) = 5x^3(2-4x)^5$

$j'(x) = 15x^2(2-4x)^5 + \overbrace{5(2-4x)^4(-4)(5x^3)}^{-100x^3(2-4x)^4}$

GCF $5x^2(2-4x)^4$

$6 - 12x - 20x$

$j'(x) = 5x^2(2-4x)^4(3(2-4x) - 20x)$

$j'(x) = \boxed{5x^2(2-4x)^4(6-32x)}$

$f(x) = x + 8x^{-2}$

8. Find an equation of the line tangent to $f(x) = x + \frac{8}{x^2}$ at the point (2, 4).

$f'(x) = 1 - 16x^{-3}$

$= 1 - \frac{16}{x^3}$

$m = -1, \ (2,4)$

$\boxed{y - 4 = -(x-2)}$

$f'(2) = 1 - \frac{16}{2^3} = 1 - \frac{16}{8} = 1 - 2 = -1$

9. Find the points on the graph of $f(x) = \frac{1}{3}x^3 + x^2 - x - 1$ at which the slope is: $f'(x) = x^2 + 2x - 1$

(a) -1	(b) 2	(c) 0
$x^2 + 2x - 1 = -1$ $x^2 + 2x = 0$ $x(x+2) = 0$ $x = 0 \quad x = -2$ $(0,-1)$ $(-2, \frac{7}{3})$	$x^2 + 2x - 1 = 2$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3, 1$ $(-3, 2)$ $(1, -\frac{2}{3})$	$x^2 + 2x - 1 = 0$ $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$ $x = -1 \pm \sqrt{2}$ $(.41, -1.22), (-2.41, 2.55)$

10. A ball is tossed in the air from a bridge and its height (y, in feet) above the ground, *t* seconds after it is thrown is given by $y = -16t^2 + 45t + 24$.

(a) What is the initial height of the bridge? What is the initial velocity of the ball?

$t = 0, \ \boxed{y_0 = 24 \text{ ft}}$ $\quad v = y' = -32t + 45 \quad \boxed{v_0 = 45 \text{ ft/sec}}$

(b) When does the ball reach the ground? What is the speed of the ball when it hits the ground?

$y = 0 = -16t^2 + 45t + 24$

$t = \frac{-45 \pm \sqrt{45^2 - 4(-16)(24)}}{2(-16)}$

$t = \frac{-45 \pm \sqrt{3561}}{-32}$

$\boxed{t = 3.271 \text{ sec}}, \cancel{-.459} \text{ sec}$

$v(t) = -32t + 45$

$v(3.271) = -59.672$ ft/sec

$\boxed{\text{Speed} = 59.672 \text{ ft/sec}}$

(c) How long does it take the ball to reach its maximum height?

At max height, $v(t) = 0$

$0 = -32t + 45$

$t = \frac{45}{32}$ $\quad \boxed{t = 1.406 \text{ sec}}$

(d) What is the maximum height the ball reaches?

$y(1.406) = \boxed{55.641 \text{ ft}}$

(e) What is the velocity function for the ball? When is its velocity -10 ft/sec?

$v(t) = -32t + 45$

$-10 = -32t + 45$

$-55 = -32t$

$t = \frac{55}{32}$ $\quad \boxed{t = 1.719 \text{ sec}}$

(f) What is the velocity 2 seconds after it's thrown?

$\boxed{v(2) = -19 \text{ ft/sec}}$

(g) What is the acceleration function for the ball? What is the acceleration 1 second after it's thrown?

$a(t) = -32$ $\quad \boxed{a(1) = -32 \text{ ft/sec}^2}$

Use the <u>limit definition</u> to find the derivative of each of the following:

11. $f(x)=\sqrt{2x-3}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{2(x+h)-3}-\sqrt{2x-3}}{h}\cdot\frac{\sqrt{2(x+h)-3}+\sqrt{2x-3}}{\sqrt{2(x+h)-3}+\sqrt{2x-3}}$$

$$=\lim_{h\to 0}\frac{\cancel{2x}+2h\cancel{-3}-(\cancel{2x}-\cancel{3})}{h\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}$$

$$=\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}$$

$$=\frac{2}{2\sqrt{2x-3}}=\boxed{\frac{1}{\sqrt{2x-3}}}$$

12. $f(x)=\dfrac{1}{x+4}$

$$f'(x)=\lim_{h\to 0}\frac{\dfrac{1\,(x+4)}{(x+h+4)(x+4)}-\dfrac{1\,(x+h+4)}{(x+4)(x+h+4)}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x}+\cancel{4}-(\cancel{x}+h+\cancel{4})}{(x+h+4)(x+4)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{-\cancel{h}}{(x+h+4)(x+4)}\cdot\frac{1}{\cancel{h}}$$

$$=\boxed{\frac{-1}{(x+4)^2}}$$

Find the derivative of the following functions:

13. $f(x)=\sqrt{9-4x^2}=(9-4x^2)^{\frac{1}{2}}$

$$f'(x)=\frac{1}{2}(9-4x^2)^{-\frac{1}{2}}(-8x)$$

$$=\frac{-\overset{4}{\cancel{8}}x}{\underset{1}{\cancel{2}}(9-4x^2)^{\frac{1}{2}}}=\boxed{\frac{-4x}{\sqrt{9-4x^2}}}$$

14. $k(x)=\dfrac{1}{x+3}=(x+3)^{-1}$

$$k'(x)=-(x+3)^{-2}(1)$$

$$=\boxed{\frac{-1}{(x+3)^2}}$$

15. $h(x)=x(2x+3)^{\frac{1}{2}}$

$$h'(x)=1(2x+3)^{\frac{1}{2}}+\frac{1}{\cancel{2}}(2x+3)^{-\frac{1}{2}}(\cancel{2})(x)$$

$$h'(x)=(2x+3)^{\frac{1}{2}}+x(2x+3)^{-\frac{1}{2}}$$

GCF $(2x+3)^{-\frac{1}{2}}$

$$h'(x)=(2x+3)^{-\frac{1}{2}}(2x+3+x)$$

$$=\boxed{\frac{3x+3}{(2x+3)^{1/2}}}$$

16. $g(x)=\dfrac{3x-2}{\cos x}$

$$g'(x)=\frac{3(\cos x)-(-\sin x)(3x-2)}{\cos^2 x}$$

$$g'=\boxed{\frac{3\cos x+(3x-2)\sin x}{\cos^2 x}}$$

17. $m(x)=2\cos(4x^2-3x)$

$$m'(x)=-2\sin(4x^2-3x)\,(8x-3)$$

$$m'(x)=\boxed{-2(8x-3)\sin(4x^2-3x)}$$

18. $n(x)=\dfrac{2x^3+3x^2-2x+4+3\pi^2x}{2x}$

$$n(x)=x^2+\frac{3}{2}x-1+2x^{-1}+\frac{3\pi^2}{2}$$

$$n'(x)=2x+\frac{3}{2}-2x^{-2}$$

$$n'(x)=\boxed{2x+\frac{3}{2}-\frac{2}{x^2}}$$

19. Given $h(x) = 3x^3(x+2)^7$, find $h'(-1)$.

$h'(x) = 9x^2(x+2)^7 + 7(x+2)^6(1)(3x^3)$

$= 9x^2(x+2)^7 + 21x^3(x+2)^6$ GCF $= 3x^2(x+2)^6$

$= 3x^2(x+2)^6(3(x+2)+7x)$

$h'(x) = 3x^2(x+2)^6(10x+6)$

$h'(-1) = 3(-1)^2(-1+2)^6(10(-1)+6)$

$= 3(1)(-4)$

$\boxed{h'(-1) = -12}$

20. Given $y = \cos x + \dfrac{3}{x^9}$, find $\dfrac{d^2y}{dx^2}$

$\dfrac{dy}{dx} = -\sin x - 27x^{-10}$

$\dfrac{d^2y}{dx^2} = -\cos x + 270x^{-11}$

$\boxed{\dfrac{d^2y}{dx^2} = -\cos x + \dfrac{270}{x^{11}}}$

21. An object is moving in such a way that the position of the object in feet at time t seconds is given by the equation $s(t) = -16t^2 + 25t + 18$.

(a) What is the average velocity of the object on the interval [0.25, 0.75]?

$\dfrac{S(.75) - S(.25)}{.75 - .25} = \dfrac{27.75 - 23.25}{.5} = \dfrac{4.5}{.5} = \boxed{9 \frac{ft}{sec}}$

(b) At what time does the object reach a maximum height?

$V(t) = 0$

$V(t) = -32t + 25 = 0$

$t = \dfrac{25}{32}$ $\boxed{t = .781 \text{ sec}}$

(c) What is the maximum height of the object?

$S(.781) = \boxed{27.766 \text{ ft}}$

(d) What is the instantaneous velocity at t = 1.25 sec?

$V(t) = -32t + 25$

$V(1.25) = \boxed{-15 \text{ ft/sec}}$

(e) How long does it take for the object to reach the ground?

$S(t) = 0 = -16t^2 + 25t + 18$

$t = \dfrac{-25 \pm \sqrt{25^2 - 4(-16)(18)}}{2(-16)}$

$t = \dfrac{-25 \pm \sqrt{1777}}{-32}$

$\boxed{t = 2.099 \text{ sec}}$, $\cancel{-.536 \text{ sec}}$

(f) What is the instantaneous velocity of the object when it hits the ground?

$V(2.099) = \boxed{-42.168 \text{ ft/sec}}$