Name Answer Key **Date**_____

Calc I H – 2.1-2.4 Review 2 **Period**_____

Multiple-Choice – Show all work when possible!

1. If $f(x)=2x^2+4$, which of the following will calculate the derivative of $f(x)$?

a.) $\dfrac{\left[2(x+\triangle x)^2+4\right]-\left(2x^2+4\right)}{\triangle x}$

b.) $\lim\limits_{\triangle x\to 0}\dfrac{\left[2x^2+4+\triangle x\right]-\left(2x^2+4\right)}{\triangle x}$

c.) $\lim\limits_{\triangle x\to 0}\dfrac{\left[2(x+\triangle x)^2+4\right]-\left(2x^2+4\right)}{\triangle x}$

d.) $\dfrac{\left(2x^2+4+\triangle x\right)-\left(2x^2+4\right)}{\triangle x}$

2. Differentiate: $y=\dfrac{1+\cos x}{1-\cos x}$

a.) -1 b.) $-2\csc x$ c.) $2\csc x$ d.) $\dfrac{-2\sin x}{(1-\cos x)^2}$

$y'=\dfrac{-\sin x(1-\cos x)-\sin x(1+\cos x)}{(1-\cos x)^2}$

$y'=\dfrac{-\sin x+\cancel{\sin x\cos x}-\sin x-\cancel{\sin x\cos x}}{(1-\cos x)^2}=\dfrac{-2\sin x}{(1-\cos x)^2}$

3. Differentiate: $y=\dfrac{3x}{x^2+1}$

a.) $\dfrac{3}{1+x^2}$ b.) $\dfrac{3}{2x}$ c.) $\dfrac{3x^2-3}{(1+x^2)^3}$ d.) $\dfrac{3(1-x^2)}{(1+x^2)^2}$

$y'=\dfrac{3(x^2+1)-2x(3x)}{(x^2+1)^2}=\dfrac{3x^2+3-6x^2}{(x^2+1)^2}=\dfrac{-3x^2+3}{(x^2+1)^2}$

$=\dfrac{3(1-x^2)}{(x^2+1)^2}$

4. Find $\dfrac{dy}{dx}$ for $y=\sqrt{x}(3x-1)=x^{\frac{1}{2}}(3x-1)$

a.) $\dfrac{9x-1}{2\sqrt{x}}$ b.) $\dfrac{9}{2}\sqrt{x}-1$ c.) $3\sqrt{x}$ d.) $\dfrac{3}{2\sqrt{x}}$

$y'=\frac{1}{2}x^{-1/2}(3x-1)+3x^{1/2}$ GCF: $x^{-1/2}$

$y'=x^{-1/2}\left(\frac{1}{2}(3x-1)+3x\right)$

$=x^{-1/2}\left(\frac{3}{2}x-\frac{1}{2}+\frac{6}{2}x\right)=x^{-1/2}\left(\frac{9x-1}{2}\right)=\dfrac{9x-1}{2\sqrt{x}}$

5. Find $\frac{dy}{dx}$ for $y=\left(x^3\right)\sqrt{x+1}$ $= x^3 \cdot (x+1)^{\frac{1}{2}}$

a.) $\frac{3x^2}{2\sqrt{x+1}}$ b.) $\frac{x^2(7x+6)}{2\sqrt{x+1}}$ c.) $3x^2\sqrt{x+1}$ d.) $\frac{7x^3+x^2}{2\sqrt{x+1}}$

$\frac{dy}{dx} = 3x^2(x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}}(1)(x^3)$ GCF: $x^2(x+1)^{-\frac{1}{2}}$

$= x^2(x+1)^{-\frac{1}{2}}\left(3(x+1)+\frac{1}{2}x\right)$

$= x^2(x+1)^{-\frac{1}{2}}\left(\frac{2}{2}3x+\frac{3}{2}2+\frac{1}{2}x\right) = x^2(x+1)^{-\frac{1}{2}}\left(\frac{7x+6}{2}\right) = \frac{x^2(7x+6)}{2\sqrt{x+1}}$

6. Find $f'(x)$ for $f(x)=\left(2x^2+5\right)^7$

a.) $7(4x)^6$ b.) $(4x)^7$ c.) $28x\left(2x^2+5\right)^6$ d.) $7\left(2x^2+5\right)^6$

$f'(x) = 7(2x^2+5)^6(4x)$

$= 28x(2x^2+5)^6$

7. Differentiate $y=\sec^2 x+\tan^2 x$.

a.) 0 b.) $\tan x+\sec^4 x$ c.) $\sec^2 x\left(\sec^2 x+\tan^2 x\right)$ d.) $4\sec^2 x\tan x$

$y' = 2\sec x(\sec x\tan x) + 2\tan x(\sec^2 x)$

$= 4\sec^2 x\tan x$

8. Find the derivative: $s(t)=\csc\frac{t}{2}$.

a.) $-\csc\frac{t}{2}\cot\frac{t}{2}$ b.) $-\frac{1}{2}\cot^2\frac{t}{2}$ c.) $-\frac{1}{2}\csc\frac{t}{2}\cot\frac{t}{2}$ d.) $\frac{1}{2}\csc\frac{t}{2}\cot\frac{t}{2}$

$S'(t) = -\csc\left(\frac{t}{2}\right)\cot\left(\frac{t}{2}\right)\cdot\left(\frac{1}{2}\right)$

$= -\frac{1}{2}\csc\left(\frac{t}{2}\right)\cot\left(\frac{t}{2}\right)$

9. Find an equation for the tangent line to the graph of $f(x)=2x^2-2x+3$ at the point where $x=1$.

a.) $y=2x-2$ b.) $y=4x^2-6x+5$ c.) $y=2x+1$ d.) $y=4x^2-6x+2$

$f'(x) = 4x-2$

$f'(1) = 4(1)-2 = 2$

$f(1) = 2(1)^2-2(1)+3$

$= 3$

$m=2$, $(1,3)$

$y-3=2(x-1)$

$y=2x+1$

10. Find all points on the graph of $f(x) = -x^3 + 3x^2 - 2$ at which there is a horizontal tangent line. $m = 0$

a.) (0, -2), (2, 2) b.) (0, -2) c.) (1, 0), (0, -2) d.) (2, 2)

$f'(x) = -3x^2 + 6x = 0$ $f(0) = -2$ $(0, -2)$

$-3x(x-2) = 0$ $f(2) = 2$ $(2, 2)$

$x = 0, \; x = 2$

11. Find point(s) on the graph of the function $f(x) = x^3 - 2$ where the slope is 3. $m = 3$

a.) (1, 3), (-1, 3) b.) (1, -1), (-1, -3) c.) $\left(\sqrt[3]{2}, 0\right)$ d.) (1, 3)

$f'(x) = 3x^2 = 3$ $f(1) = -1$ $(1, -1)$

$x^2 = 1$ $f(-1) = -3$ $(-1, -3)$

$x = \pm 1$

12. Find the value of the derivative of the function $f(t) = \dfrac{t^3 + 2}{t}$ at the point (-2, 3).

a.) -4.5 b.) -3.5 c.) 12 d.) $-\dfrac{11}{16}$

$f'(t) = \dfrac{3t^2(t) - 1(t^3+2)}{t^2} = \dfrac{3t^3 - t^3 - 2}{t^2}$

$= \dfrac{2t^3 - 2}{t^2}$

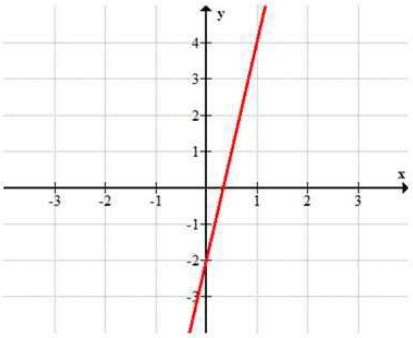
$f'(-2) = \dfrac{2(-2)^3 - 2}{(-2)^2} = \dfrac{-18}{4}$

$= -4.5$

13. The graph at the right represents the graph of the derivative of which of the following functions?

a.) $f(x) = 2x^2 + 1$ b.) $f(x) = 2x - 3$ c.) $f(x) = 3x^2 - 2x - 1$ d.) $f(x) = x^3 + x^2$ $f'(x) = 3x^2 - 2x$

$f'(x) = 4x$ $f'(x) = 2$ $f'(x) = 6x - 2$



14. The position function for a particular object is $s = -\dfrac{35}{2}t^2 + 58t + 91$. Which statement is true?

a.) The initial velocity is -35. b.) The velocity is a constant.

c.) The velocity at time t =1 is 23. ✓ d.) The initial position is $-\dfrac{35}{2}$

$s' = -35t + 58$ $s'(0) = 58$

15. Find an equation of the tangent line to the graph of $f(\theta)=\tan\theta$ at the point $\left(\frac{\pi}{4},1\right)$.

a.) $4x-4y=\pi-4$ b.) $4\sqrt{2}x-4y=\pi-4$ c.) $4x-2y=\pi-2$ d.) $y=x$

$f'(\theta)=\sec^2\theta$

$f'\left(\frac{\pi}{4}\right)=\left(\sec\left(\frac{\pi}{4}\right)\right)^2$

$=(\sqrt{2})^2=2$

$m=2$

$y-1=2\left(x-\frac{\pi}{4}\right)$

$2(y-1)=\left(2x-\frac{\pi}{2}\right)2$

$2y-2=4x-\pi$

$+\pi \quad -2y \quad +\pi \quad -2y$

$\pi-2=4x-2y$

16. Find the derivative: $f(x)=\frac{1}{\sqrt[3]{3-x^3}}$ $=(3-x^3)^{-1/3}$

a.) $\frac{-1}{3\left(3-x^3\right)^{\frac{4}{3}}}$ b.) $\frac{x^2}{\left(3-x^3\right)^{\frac{4}{3}}}$ c.) $\frac{-x^2}{\left(3-x^3\right)^{\frac{2}{3}}}$ d.) $\frac{-x^2}{\left(3-x^3\right)^{\frac{4}{3}}}$

$f'(x)=-\frac{1}{3}(3-x^3)^{-4/3}(-3x^2)$

$=(3-x^3)^{-4/3}(x^2)=\frac{x^2}{(3-x^3)^{4/3}}$

17. Find the derivative: $f(\theta)=\sqrt{\sin 2\theta}$ $=(\sin(2\theta))^{1/2}$

a.) $\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}$ b.) $\sqrt{\sec 2\theta}$ c.) $\frac{\cos 2\theta}{2\sqrt{\sin 2\theta}}$ d.) $\cos\theta$

$f'(\theta)=\frac{1}{2}(\sin(2\theta))^{-1/2}(\cos(2\theta))(2)$

$=\frac{\cos(2\theta)}{\sqrt{\sin 2\theta}}$

18. An object is thrown straight down from the top of a 220-ft building with an initial velocity of 26 ft/sec.

a.) Write the position function for the movement described.

$s(t)=-16t^2-26t+220$

b.) What is the velocity after 2 seconds?

$v(t)=s'(t)=-32t-26$ $v(2)=-90$ ft/sec

c.) What is the acceleration after 2 seconds?

$a(t)=s''(t)=-32$ ft/sec^2