2.1 day 1 - Limit Definition of Derivative

10/18/18

Homework:

- 2.1 A
- Quiz 2.1 Thursday 10/25 or Friday 10/26

Math tells us three of the saddest love stories.

Tangent lines who had one chance to meet and then parted forever.

Parallel lines who were never meant to meet.

Asymptotes who can get closer and closer but will never be together.

Objective: Use the limit definition to find the derivative of a function.

Do Now:

then

$f(x+h)$

$$f(x) = 2x + 1$$

$$\frac{2(x+h)+1-(2x+1)}{h}$$

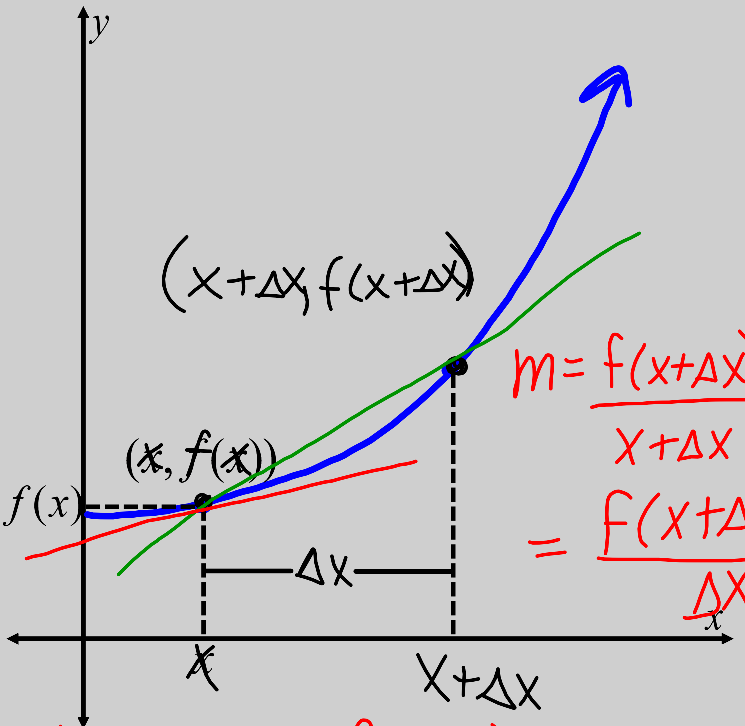
$$\frac{\cancel{2x}+2h\cancel{+1}-\cancel{2x}\cancel{-1}}{h} = \frac{2h}{h} = \boxed{2}$$

$$f(x) = \frac{5}{x+1} \qquad \frac{f(x+h)-f(x)}{h}$$

1. $$\frac{\left(\frac{5}{x+h+1} - \frac{5}{x+1}\right)(x+h+1)(x+1)}{(h)(x+h+1)(x+1)}$$

2. $$\frac{5(x+1)-5(x+h+1)}{h(x+h+1)(x+1)} = \frac{\cancel{5x}+\cancel{5}-\cancel{5x}-5h-\cancel{5}}{h(x+h+1)(x+1)}$$

3. $$\frac{-5\cancel{h}}{\cancel{h}(x+h+1)(x+1)} = \boxed{\frac{-5}{(x+h+1)(x+1)}}$$



$$m = \frac{f(x+\Delta x)-f(x)}{x+\Delta x - x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

he **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

rovided the limit exists. For all x for which this limit exists, f' is a function

NITION Derivative

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$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}, \qquad (1)$$

ed the limit exists.

Notation

There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are:

y'	"*y* prime"	Nice and brief, but does not name the independent variable.
$\dfrac{dy}{dx}$	"*dy dx*" or "the derivative of *y* with respect to *x*"	Names both variables and uses *d* for derivative.
$\dfrac{df}{dx}$	"*df dx*" or "the derivative of *f* with respect to *x*"	Emphasizes the function's name.
$\dfrac{d}{dx} f(x)$	"*d dx* of *f* at *x*" or "the derivative of *f* at *x*"	Emphasizes that *differentiation* is an operation performed on *f*.

Differentiate - verb
Derivative - noun

Find the derivative of $f(x) = 3 + \frac{2}{3}x.$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3+\frac{2}{3}(x+\Delta x) - \left(3+\frac{2}{3}x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3}+\cancel{\frac{2}{3}x}+\frac{2}{3}\Delta x - \cancel{3} - \cancel{\frac{2}{3}x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2}{3}\Delta x}{\Delta x} = \boxed{\frac{2}{3}}$$

Find $f'(x)$ given $f(x) = 4x^2 + 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$4(x^2+2xh+h^2)$$

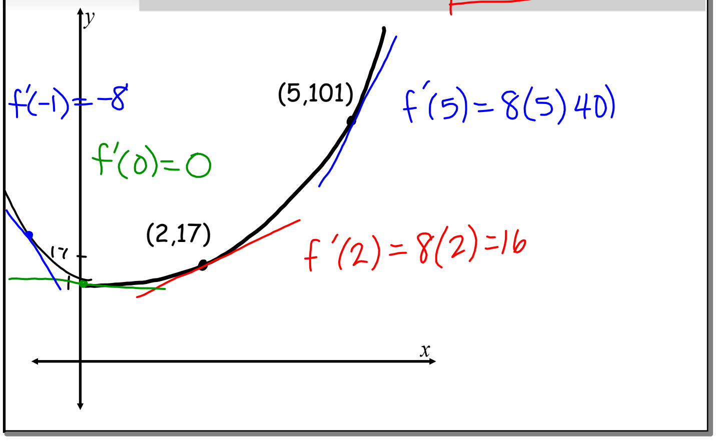
$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2+1-(4x^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4x^2}+8xh+4h^2+\cancel{1}-\cancel{4x^2}-\cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(8x+4h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (8x+4h) = 8x$$

$$\boxed{f'(x) = 8x}$$



Find the slope of the tangent line to

$f(x) = x^3 - 2$ at the point (3,25).

$$f'(x) = \lim_{h \to 0} \frac{\overbrace{(x+h)^3 - 2}^{f(x+h)} - \overbrace{(x^3-2)}^{f(x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-2} \cancel{-x^3} \cancel{+2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} =$$

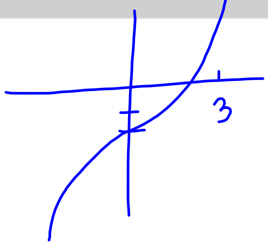
$$= \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$$

$$\boxed{f'(x) = 3x^2}$$

(3,25)

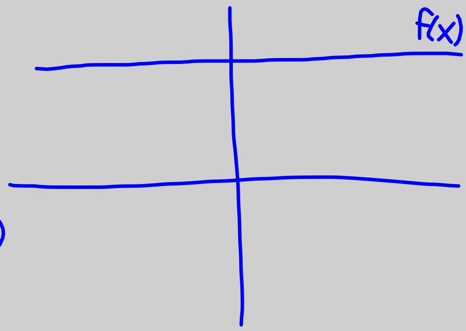
$f'(3) = 3(3)^2 = 27$

$f(x) = x^3 - 2$



Based on our discussions today what would the derivative of the following function be, and why?

$$f(x) = 5$$



$f'(x) = 0$