Calculus H
Quiz 2.1 Review – Finding Derivatives using Limit Definition

Name ______________________
Date ______________________

Given $y = 7x^2 + 2x - 71$,

1) a. Find $\frac{dy}{dx}$ by using the definition of the derivative.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{7(x+h)^2 + 2(x+h) - 7 - (7x^2+2x-7)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{7x^2} + 14xh + 7h^2 + \cancel{2x} + 2h \cancel{-7} \cancel{-7x^2} \cancel{-2x} \cancel{+7}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(14x+7h+2)}{\cancel{h}} = \lim_{h\to 0}(14x+7h+2)$$

$$= \boxed{14x+2}$$

b. Find $y'(0)$.

$$y'(0) = 14(0) + 2 = \boxed{2}$$

c. Write the equation of the tangent line at $x = 0$.

c. $y = 7x^2 + 2x - 7$

$y(0) = 7(0)^2 + 2(0) - 7 = -7$

$(0, -71)$ $m = y'(0)$

$$\boxed{\begin{array}{l} y+7 = 2(x-0) \\ y+7 = 2x \end{array}}$$

d. Write the equation of a tangent line that would have a slope of 30.

$\frac{dy}{dx} = 14x + 2 = 30$

$14x = 28$

$x = 2$

$y(2) = 7(2)^2 + 2(2) - 7 = 25$

$(2, 25)$ $m = 30$

$\boxed{y - 25 = 30(x-2)}$

e ~~d~~. Does the function have a greater rate of change at x = -2 or at x = 1? Explain your reasoning.

$y'(-2) = 14(-2) + 2 = -26$

$y'(1) = 14(1) + 2 = 16$

Since the slope is steeper at $x = -2$, the rate of change at $x = -2$ is greater than rate of change at $x = 0$.

4) Given $y = \frac{4}{2x+3}$, find $\frac{dy}{dx}$ by using the limit definition of the derivative.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{4(2x+3)}{(2(x+h)+3)(2x+3)} - \frac{4}{(2x+3)}\frac{(2(x+h)+3)}{(2(x+h)+3)}}{h} = \lim_{h\to 0} \frac{\cancel{8x} + \cancel{12} - \cancel{8x} - 8h - \cancel{12}}{h(2(x+h)+3)(2x+3)}$$

$$= \lim_{h\to 0} \frac{-8\cancel{h}}{\cancel{h}(2(x+h)+3)(2x+3)} = \lim_{h\to 0} \frac{-8}{(2(x+h)+3)(2x+3)}$$

$$= \boxed{\frac{-8}{(2x+3)^2}}$$

5) Given $y=\sqrt{2x-3}$, find y' by using the limit definition of the derivative.

$$y'=\lim_{h\to 0}\frac{\sqrt{2(x+h)-3}-\sqrt{2x-3}}{h}\cdot\frac{\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}{\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}$$

$$=\lim_{h\to 0}\frac{2x+2h-3-(2x-3)}{h\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}=\lim_{h\to 0}\frac{\cancel{2x}+2h-\cancel{3}-\cancel{2x}+\cancel{3}}{h\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}$$

$$=\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(x+h)-3}+\sqrt{2x-3}\right)}=\lim_{h\to 0}\frac{2}{\sqrt{2(x+h)-3}+\sqrt{2x-3}}$$

$$=\frac{\cancel{2}}{\cancel{2}\sqrt{2x-3}}=\boxed{\frac{1}{\sqrt{2x-3}}}$$

Find the first derivative by using the limit definition: (#6 – #9)

6) $y=4x^2-5x+4$

$$y'=\lim_{h\to 0}\frac{4(x+h)^2-5(x+h)+4-(4x^2-5x+4)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{4x^2}+8xh+4h^2-\cancel{5x}-5h+\cancel{4}-\cancel{4x^2}+\cancel{5x}-\cancel{4}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(8x+4h-5)}{\cancel{h}}$$

$$=\lim_{h\to 0}(8x+4h-5)=\boxed{8x-5}$$

7) $y=x^3+2$

$$y'=\lim_{h\to 0}\frac{(x+h)^3+2-(x^3-2)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{2}-\cancel{x^3}+\cancel{2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(3x^2+3xh+h^2)=\boxed{3x^2}$$

8) $y=\dfrac{-4}{x+9}$

$$y'=\lim_{h\to 0}\frac{\frac{-4(x+9)}{(x+h+9)(x+9)}-\frac{-4(x+h+9)}{(x+9)(x+h+9)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\cancel{-4x}-\cancel{36}+\cancel{4x}+4h+\cancel{36}}{(x+h+9)(x+9)}}{h}$$

$$=\lim_{h\to 0}\frac{4\cancel{h}}{(x+h+9)(x+9)}\cdot\frac{1}{\cancel{h}}$$

$$=\lim_{h\to 0}\frac{4}{(x+h+9)(x+9)}=\boxed{\frac{4}{(x+9)^2}}$$

9) $y=\sqrt{x-5}$

$$y'=\lim_{h\to 0}\frac{\left(\sqrt{x+h-5}-\sqrt{x-5}\right)}{h}\cdot\frac{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}{\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$y'=\lim_{h\to 0}\frac{x+h-5-(x-5)}{h\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$y'=\lim_{h\to 0}\frac{\cancel{x}+h-\cancel{5}-\cancel{x}+\cancel{5}}{h\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-5}+\sqrt{x-5}\right)}=\lim_{h\to 0}\frac{1}{\sqrt{x+h-5}+\sqrt{x-5}}$$

$$=\boxed{\frac{1}{2\sqrt{x-5}}}$$

10) Find an equation of the tangent line to the graph of $f(x)=x^2+2x+1$ at the point $(-3,4)$.

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2+2(x+h)+1-(x^2+2x+1)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2+\cancel{2x}+2h+\cancel{1}-\cancel{x^2}-\cancel{2x}-\cancel{1}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h+2)}{\cancel{h}}=\lim_{h\to 0}(2x+h+2)=2x+2$$

$(-3,4)$ $\quad m=f'(-3)=2(-3)+2$
$=-4$

$$\boxed{y-4=-4(x+3)}$$

11) Given $f(x)=\dfrac{12x}{x+3}$,

$$f'(x)=\lim_{h\to 0}\frac{\dfrac{\overbrace{(12(x+h))(x+3)}^{x^2+3x+xh+3h}}{((x+h)+3)(x+3)}-\dfrac{12x(x+h+3)}{(x+3)(x+h+3)}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{\cancel{12x^2}+\cancel{36x}+\cancel{12xh}+36h-\cancel{12x^2}-\cancel{12xh}-\cancel{36x}}{((x+h)+3)(x+3)}}{h}$$

$$=\lim_{h\to 0}\frac{36\cancel{h}}{((x+h)+3)(x+3)}\cdot\frac{1}{\cancel{h}}=\lim_{h\to 0}\frac{36}{((x+h)+3)(x+3)}=\frac{36}{(x+3)^2}$$

a) Find $f'(0)$.

$f'(0)=\dfrac{36}{3^2}=\dfrac{36}{9}$

$\boxed{f'(0)=4}$

b) Find $f'(-2)$.

$f'(-2)=\dfrac{36}{(-2+3)^2}=\boxed{36}$

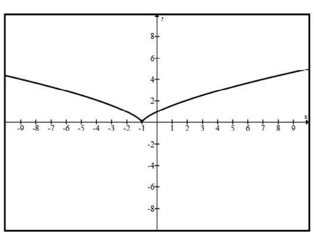
c) Write an equation of the tangent line to *f* at the 2 points given in parts a and b.

$f(0)=0$
$(0,0)$ $\quad m=f'(0)=4$
$\boxed{y=4x}$

$f(-2)=\dfrac{-24}{1}=-24$
$(-2,-24)$ $\quad m=f'(-2)=36$
$\boxed{y+24=36(x+2)}$

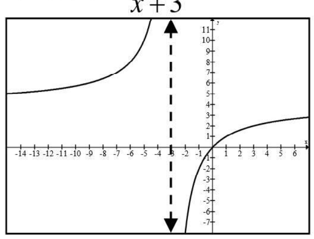
12) Use interval notation to write the x-values at which each function is differentiable given the following graphs.

a) $f(x)=(x+1)^{\frac{2}{3}}$



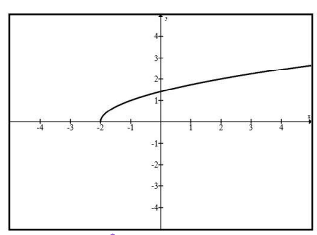
$(-\infty,-1)\cup(-1,\infty)$

b) $g(x)=\dfrac{4x}{x+3}$



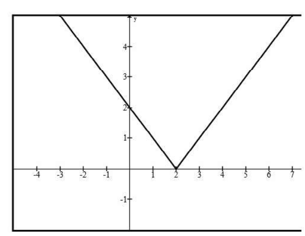
$(-\infty,-3)\cup(-3,\infty)$

c) $h(x)=\sqrt{x+2}$



$(-2,\infty)$

d) $p(x)=|x-2|$



$(-\infty,2)\cup(2,\infty)$

13) What is the derivative of $f(x)=-3$?

$y=-3$ is a horizontal line with slope $m=0$. $\therefore f'(x)=0$