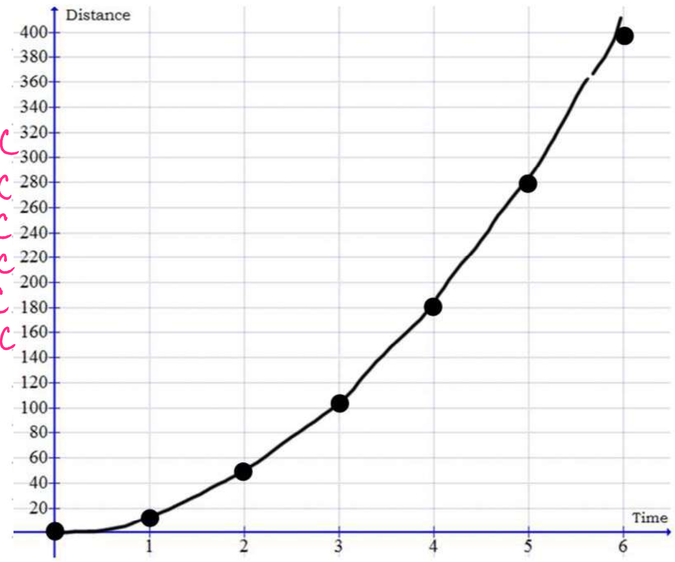
Name______________________________ Date______________

Calc I H – 2.2 day 4 & 5 – Position, Velocity & Acceleration Period______

Do Now:
A car company needs more data of a car that they are developing so they decide to perform a test run. Below is a chart representing the distance, in feet, traveled by the car during the first six seconds of its test run. Use the data to provide a sketch.

Time (sec)	Distance (ft)
0	0
1	16
2	52
3	108
4	184
5	280
6	396

> 16 ft/sec
> 36 ft/sec
> 56 ft/sec
> 76 ft/sec
> 96 ft/sec
> 116 ft/sec



$d = r \cdot t$

$r = \frac{d}{t}$

1. Using the data above, what is the average velocity of the car during the first six seconds of the test? The first two seconds? The last two seconds? Between 3 and 4 seconds?

from 0-6 sec $\frac{396-0}{6-0} = 66$ ft/sec

from 0-2 sec $\frac{52-0}{2-0} = 26$ ft/sec

from 4-6 sec $\frac{396-184}{6-4} = 106$ ft/sec

from 3-4 sec $\frac{184-108}{4-3} = 76$ ft/sec

2. Using the data above, give an estimate of the *instantaneous* velocity of the car at 3.5 seconds.

Approx avg velocity from 3-4 sec: 76 ft/sec

3. How does average velocity and instantaneous velocity relate to your sketch above?

Slope of secant line

slope of tangent line

4. Is the car accelerating or decelerating during its test run? Explain graphically.

Accelerating. The velocity is increasing over time.

5. The company discovers that the data models the function $s(t) = 10t^2 + 6t$, where s (distance in feet) is a function of t (time in seconds) for $0 \le t \le 6$. Using this fact, can you improve upon your previous estimate for the car's instantaneous velocity at $t = 3.5$?

$V(t) = S'(t) = 20t + 6$

$V(3.5) = 20(3.5) + 6 = 76$ ft/sec

Class Notes:

Using calculus, we can define the relationship between the position function, velocity function and acceleration function of an object as follows:

Position $- s(t)$

Average velocity $- \Delta V = \frac{\Delta s}{\Delta t}$

(Slope of secant line)

Instantaneous velocity $- v(t) = s'(t)$

(Slope of tangent line)

Speed $= |v(t)|$

acceleration $- a(t) = v'(t) = s''(t)$

Examples:

1. A ball is tossed into the air from a bridge and its height, y (in feet), above the ground, t seconds after it is thrown is given by: $s(t) = -16t^2 + 50t + 36$

    a. What is the object's initial position? Initial velocity?

    $t=0$ $\boxed{s(0) = 36 \text{ ft}}$

    $v(t) = s'(t) = -32t + 50$

    $\boxed{v(0) = 50 \text{ ft/sec}}$

    b. When does the object hit the ground?

    $s(t) = 0 = -16t^2 + 50t + 36$ $\quad t = \frac{-50 \pm \sqrt{(50)^2 - 4(-16)(36)}}{2(-16)}$ $\quad t = \cancel{-.603}, 3.728$

    $\boxed{t = 3.728 \text{ sec}}$

    c. What is the object's velocity when it hits the ground?

    $v(3.728) = -32(3.728) + 50 = \boxed{-69.311 \text{ ft/sec}^2}$

    69.311 ft/sec² down

    d. When does the object reach its maximum height?

    $v(t) = 0 = -32t + 50$

    $t = \frac{50}{32} = \boxed{1.5625 \text{ sec}}$

    e. What is the maximum height of the object?

    $s(1.5625) = 75.0625$ ft.

    f. When is the object 40 ft high?

    $s(t) = 40 = -16t^2 + 50t + 36$

    $0 = -16t^2 + 50t - 4$

    $t = \frac{-50 \pm \sqrt{50^2 - 4(-16)(-4)}}{2(-16)}$

    $\boxed{t = .082, 3.043 \text{ sec}}$

    g. Moving at 20 ft/sec down?

    $v(t) = -20 = -32t + 50$

    $-70 = -32t$

    $\boxed{t = 2.1875 \text{ sec}}$

2.2 WORKING WITH THE POSITION FUNCTION

The position of a free-falling object (neglecting air resistance) under the influence of gravity can be represented by the equation:

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

where s_0 is the *initial height* of the object
v_0 is the *initial velocity* of the object
g is the *acceleration due to gravity*

On Earth, the value of the gravitational constant g is approximately -32 ft/sec^2 or -9.8 m/sec^2.
(Think about this: Why is it negative?)

Therefore:

When measuring in **feet**, use the function: $s(t) = -16t^2 + v_0 t + s_0$

When measuring in **meters**, use the function: $s(t) = -4.9t^2 + v_0 t + s_0$

HELPFUL HINTS

1) To find the *initial height*, s_0:
   (a) Let $t = 0$ in $s(t)$
   (b) Solve for s_0

2) To find the *initial velocity*, v_0:
   (a) Find $s'(t)$
   (b) Let $t = 0$ in $s'(t)$
   (c) Solve for v_0

3) To find the *time*, *t*, it takes an object to reach its **maximum height**:
   (a) Find $s'(t)$
   (b) Set $s'(t) = 0$
   (c) Solve for *t*

4) To find the **maximum height**:
   (a) Find *t* (See hint #3)
   (b) Substitute the *t* value back into the position function

5) To find the *time*, *t*, it will take for an object to **hit the ground**:
   (a) Set $s(t) = 0$
   (b) Solve for *t*

2. A rocket thrown upward from the top of a building follows a path given by $s(t) = -16t^2 + 48t + 20$.

   a. Find the height of the building.

   $S(0) = \boxed{20 \text{ ft.}}$

   b. When will the rocket hit the ground?

   $S(t) = 0 = -16t^2 + 48t + 20$

   $t = \dfrac{-48 \pm \sqrt{48^2 - 4(-16)(20)}}{2(-16)} = \cancel{-.371}, 3.371$

   $\boxed{t = 3.371 \text{ sec}}$

   c. What time does the rocket reach its maximum height?

   $V(t) = 0$

   $V(t) = S'(t) = -32t + 48 = 0$

   $t = \dfrac{48}{32}$

   $\boxed{t = 1.5 \text{ sec}}$

   d. What is the maximum height the rocket reached?

   $\boxed{S(1.5) = 56 \text{ ft.}}$

   e. What was the initial velocity?

   $\boxed{V(0) = 48 \text{ ft/sec}}$

   f. What is the velocity after 2 seconds?

   $\boxed{V(2) = -16 \text{ ft/sec}}$ heading down!

   g. What is the average velocity in the first 2 seconds? $\dfrac{S(2) - S(0)}{2 - 0} = \dfrac{52 - 20}{2} = \boxed{16 \text{ ft/sec}}$

3. A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is the velocity after 3 seconds? What is the velocity after falling 108 feet? Use the position function $s(t) = -16t^2 + v_0 t + s_0$

   s_0 v_0

$s(t) = -16t^2 - 22t + 220$

$V(t) = s'(t) = -32t - 22$

$\boxed{V(3) = -118 \text{ ft/sec}}$

$s(t) = 108 = -16t^2 - 22t + 220$

$0 = -16t^2 - 22t + 112$

$t = \dfrac{22 \pm \sqrt{(-22)^2 - 4(-16)(112)}}{2(-22)}$

$t = \cancel{-2.488}, 1.488 \text{ sec}$

$\boxed{V(1.488) = -69.619 \text{ ft/sec}}$

4. A dynamite blast propels a heavy rock straight up with a launch velocity of 160ft/sec (about 109 mph). It reaches a height of $s(t)=160t-16t^2$ ft after t seconds.

   a. How high does the rock go?

   $v(t)=s'(t)=160-32t$

   $v(0)=0=160-32t$

   $t=\frac{160}{32}=5$ sec

   $\boxed{S(5)=400 \text{ ft.}}$

   b. What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

   $S(t)=256=160t-16t^2$

   $\frac{16t^2-160t+256=0}{16}$

   $t^2-10t+16=0$

   $(t-2)(t-8)=0$

   $t=2,8$ sec

   $\boxed{\begin{array}{l} v(2)=96 \text{ ft/sec} \\ v(8)=-96 \text{ ft/sec} \end{array} \text{ speed } 96 \text{ ft/sec}}$

   c. What is the acceleration of the rock at any time t during its flight (after the blast)?

   $\boxed{a(t)=v'(t)=-32 \text{ ft/s}^2}$

   d. When does the rock hit the ground?

   $S(t)=160t-16t^2=0$

   $16t(10-t)=0$

   $\cancel{t=0}$ $\boxed{t=10 \text{ sec}}$

5. A rock thrown vertically upward from the surface of the moon at a velocity of $s(t)=24t-0.8t^2$ reaches a height of s meters in t seconds.

   a. Find the rock's velocity and acceleration as functions of time. (The acceleration in this case is the acceleration of gravity on the moon.)

   $v(t)=s'(t)=24-1.6t$

   $a(t)=v'(t)=-1.6$

   b. How long did it take the rock to reach its highest point?

   $v(t)=0=24-1.6t$

   $t=\frac{24}{1.6}=\boxed{15 \text{ sec}}$

   c. How high did the rock go?

   $\boxed{S(15)=180 \text{ ft.}}$

6. On the moon, an object is dropped straight down into a crater 150 m deep. The height of the object from the bottom of the crater, as it falls, is given by: $h(t) = -0.8t^2 + 150$ where t is measured in seconds.
   a. How long will it take this object to reach the ground at the bottom of the crater?

$h(t) = 0 = -0.8t^2 + 150$

$t^2 = \frac{150}{0.8} = 187.5$

$t = \pm 13.693$

$\boxed{t = 13.693 \text{ sec}}$

   b. What is the falling object's maximum speed?

$v(t) = h'(t) = -1.6t$

Since the object is in free fall, max speed will occur on impact with ground.

$v(13.698) = -21.909$ m/s

$\boxed{\text{Speed} = 21.909 \text{ m/s}}$

7. A particle is moving vertically so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.
   a. Find the average velocity of the particle during the first 3 seconds.

$\frac{s(3) - s(0)}{3 - 0} = \frac{0 - 3}{3} = \boxed{-1 \text{ m/s}}$

   b. Find the instantaneous velocity of the particle when $t = 3$.

$v(t) = s'(t) = 2t - 4$

$v(3) = \boxed{2 \text{ m/s}}$

   c. At what time does the particle change direction? What direction was it travelling prior to this time? After this time?

$t=3$

$v(t) = 0 = 2t - 4$

$\boxed{t = 2 \text{ sec}}$

$t = 1$ sec

$\left. \begin{array}{l} v(1) = -2 \\ v(1) < 0 \end{array} \right\}$ moving down before 2 sec

$\left. \begin{array}{l} v(3) = 2 \\ v(3) > 0 \end{array} \right\}$ moving up after 2 sec

8. On three different planets, the following functions express the height of an object that is thrown downward and is falling 200 meters where t is measured in seconds.

Neptune: $n(t) = -5.88t^2 - 8t + 200$
Mars: $m(t) = -1.85t^2 - 8t + 200$
Venus: $w(t) = -4.45t^2 - 8t + 200$

a. Which of the planets has the strongest gravitational pull? Justify your answer.

Neptune! It's gravitational Constant is -5.88 which is the greatest.

b. What is the initial speed of the falling object?

$V_n(t) = n'(t) = -11.76t - 8$ $\quad V_m(t) = m'(t) = -3.7t - 8$ $\quad V_w(t) = -8.9t - 8$

$V_n(0) = -8$ m/s $\quad V_m(0) = -8$ m/s $\quad V_w(0) = -8$ m/s

$\boxed{\text{Initial Speed} = 8 \text{ m/s}}$

c. On which planet will the impact speed be the greatest and why? What is the speed at impact with the ground on each planet?

Neptune

$n(t) = 0 = -5.88t^2 - 8t + 200$

$t = \dfrac{8 \pm \sqrt{8^2 - 4(-5.88)(200)}}{2(-5.88)}$

$t = \dfrac{8 \pm \sqrt{4768}}{-11.76} = \cancel{-6.552},\ 5.191$

$V_n(5.191) = -69.046$ m/s

Mars

$m(t) = 0 = -1.85t^2 - 8t + 200$

$t = \dfrac{8 \pm \sqrt{8^2 - 4(-1.85)(200)}}{2(-1.85)}$

$t = \dfrac{8 \pm \sqrt{1544}}{-3.7} = \cancel{-12.782},\ 8.458$

$V_m(8.458) = -39.294$ m/s

Venus

$w(t) = 0 = -4.45t^2 - 8t + 200$

$t = \dfrac{8 \pm \sqrt{8^2 - 4(-4.45)(200)}}{2(-4.45)}$

$t = \dfrac{8 \pm \sqrt{3624}}{-8.9} = \cancel{-7.663},\ 5.865$

$V_w(5.865) = -60.200$ m/s

Speed on Neptune is the greatest because it has the greatest gravitational pull.

d. In order to ensure a soft enough landing a parachute will need to deploy at the moment the object is traveling -30m/s. How much sooner will the parachute have to deploy on Venus than on Mars?

Venus

$V_w(t) = -8.9t - 8 = -30$

$-8.9t = -22$

$t = 2.472$ sec

Mars

$V_m(t) = -3.7t - 8 = -30$

$-3.7t = -22$

$t = 5.946$ sec

$5.946 - 2.472 = 3.474$

Parachute will deploy 3.474 sec sooner on Venus than on Mars.

9. In science class water balloons are tossed upward with an initial velocity of 30 ft/s from the roof of the school 25 feet high and then allowed to drop to the ground below the building. This can be modeled by the function $h(t) = -16t^2 + 30t + 25$.

   a. What is the maximum height of the water balloon?

$V(t) = 0$

$v(t) = h'(t) = -32t + 30 = 0$

$t = .9375$ sec

$\boxed{h(.9375) = 39.0625 \text{ ft}}$

   b. At what time(s) will the water balloon be traveling at a speed of 20 ft/s?

velocity $= \pm 20 \frac{ft}{sec}$

$V(t) = -32t + 30 = 20$

$-32t = -10$

$\boxed{t = .3125 \text{ sec}}$

$V(t) = -32t + 30 = -20$

$-32t = -50$

$\boxed{t = 1.5625 \text{ sec}}$

   c. What will the velocity of the water balloon be when it passes the roof of the building on its way to the ground?

$h(t) = -16t^2 + 30t + 25$

$25 = -16t^2 + 30t + 25$

$0 = -16t^2 + 30t$

$0 = t(-16t + 30)$

$\cancel{t = 0}$ initial

$h(0) = 25 \Rightarrow$ Initial height

$t = \frac{30}{16} = 1.875$ sec

$\boxed{V(1.875) = -30 \text{ ft/sec}}$

   d. What will the velocity be when the water balloon hits the ground?

$h(t) = 0 = -16t^2 + 30t + 25$

$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(25)}}{2(-16)}$

$t = \frac{-30 \pm \sqrt{2500}}{-32} = \cancel{-.625}, 2.5$ sec

$\boxed{V(2.5) = -50 \text{ ft/sec}}$

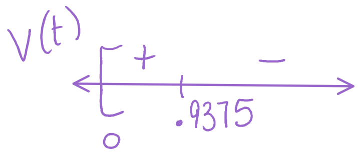
   e. At what time will the water balloon change direction?

when $v(t)$ changes sign.

$V(t) = -32t + 30 = 0$

$t = 30/32$

$t = .9375$ sec



$V(t) > 0$ before $t = .9375 \Rightarrow$ moving right

$V(t) < 0$ after $t = .9375 \Rightarrow$ moving left