2.3 day 1 - Product and Quotient Rule

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"Who is putting all the Math books in the Horror section?"

Homework:
- Section 2.3A
- Quiz 2.1-2.4 - Tuesday, 10/16

Objective:

Find the derivative of a function using the Product and Quotient Rules.

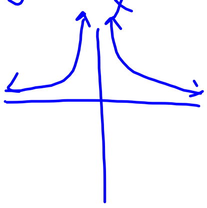
Do Now:

Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

$m=0$ a.) $y=\dfrac{1}{x^2}=x^{-2}$ b.) $y=x+2\sin x,\quad 0\le x<2\pi$

What do you know about the graph of the function where the horizontal tangent line occurs?

$y'=\dfrac{-2}{x^3}\neq 0$



$y'=1+2\cos x=0$

$\cos x=-\frac{1}{2}$

$x=\frac{2\pi}{3},\frac{4\pi}{3}$

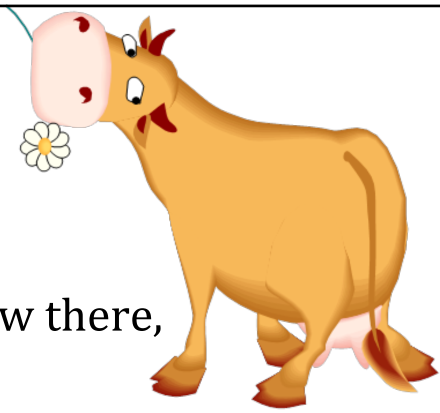
$\left(\frac{2\pi}{3},\frac{2\pi}{3}+\sqrt{3}\right),\left(\frac{4\pi}{3},\frac{4\pi}{3}-\sqrt{3}\right)$

Product Rule:

$$\frac{d}{dx}\big(f(x)g(x)\big)=f'(x)g(x)+f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$$



Low D-High minus High D-Low
E-I-E-I-O!
Draw a line, low squared below
E-I-E-I-O!
With a Low D-High here & a high D-Low there,
Low D-High, High D-Low
Underneath goes Low Low

Mrs. Centimole takes full responsibility
for this song for the Quotient Rule!

Differentiate $m(x)=\dfrac{x^2-1}{x^2+1}$.

f

g

$$m'(x)=\frac{2x(x^2+1)-(x^2-1)(2x)}{(x^2+1)^2}$$

$$m'(x)=\frac{2x(\cancel{x^2}+1-\cancel{x^2}+1)}{(x^2+1)^2}$$

$$m'(x)=\frac{4x}{(x^2+1)^2}$$

Find $p'(x)$ if $p(x)=\overset{f}{(x^2+3)}\overset{g}{(x^3+e)}$.

$$p'(x)=\overset{f'\cdot g}{2x(x^3+e)}+\overset{f\cdot g'}{(x^2+3)(3x^2)}$$

$$=2x^4+2ex+3x^4+9x^2$$

$$=5x^4+2ex+9x^2$$

Find an equation of the tangent line to the graph of *s(x)* at the given point.

$$s(x)=\frac{3-\overset{x^{-1}}{(1/x)}}{x+5},\quad(-1,1)$$

$$S(x)=\frac{\left(3-\frac{1}{x}\right)}{(x+5)}\cdot\frac{x}{x}=\frac{3x-1}{x^2+5x}$$

$$S'(x)=\frac{3(x^2+5x)-(2x+5)(3x-1)}{(x^2+5x)^2}$$

$(-1,1)$

$$S'(x)=\frac{3x^2+15x-(6x^2+13x-5)}{(x^2+5x)^2}$$

$m=p'(-1)$

$$S'(x)=\frac{-3x^2+2x+5}{(x^2+5x)^2}$$

$$p'(-1)=\frac{-3-2+5}{16}$$

$p'(-1)=0$

$\boxed{y=1}$

Let $y = uv$ be the product of the function u and v.

Find $\boxed{y'(1)}$ if $u(1) = 2,\ u'(1) = 3, v(1) = -1,$ and $v'(1) = 1.$

$y' = u'v + uv'$

$y'(1) = 3(-1) + 2(1)$

$= -1$

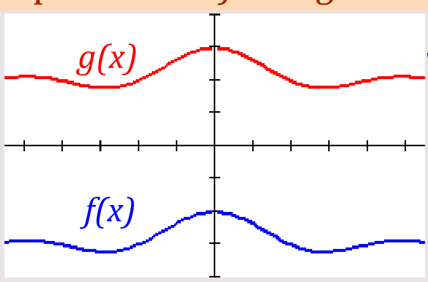
Find the derivative of each function.

$q(x) = \dfrac{\sin x - 3x}{x}$ $\qquad$ $r(x) = \dfrac{\sin x + 2x}{x}$

$g'(x) = \dfrac{x\cos x - \sin x}{x^2}$ $\qquad$ $r'(x) = \dfrac{x\cos x - \sin x}{x^2}$

Explain the relationship between f and g based on the derivatives.

$g(x) = \dfrac{\sin x}{x} - 3$



$r(x) = \dfrac{\sin x}{x} + 2$

If $c(x)=\dfrac{x^2}{1-x^3}$, find $\dfrac{d}{dx}c(x)$ $= \dfrac{x^4+2x}{(1-x^3)^2}$

AP Calculus AB **Section 2.3:** Product and Quotient Rules **Day 1**

Find the derivative of each of the following functions. Be sure to use correct notation.

1. $f(x)=x^{2/3}+3x^{1/3}+2$
2. $y=\dfrac{6x-2}{x^3}$
3. $f(x)=2x+3x^2\sin x$
4. $h(x)=2\sqrt{x}+3x^2\cos x$
5. $g(x)=(x+3)(2x-1)-4\cos x$
6. $y=(x^2-1)(3x^2+2x)$
7. $g(x)=2x^2\sin x$
8. $f(x)=\sqrt{x}(x+1)$
9. $f(x)=\dfrac{1}{x}\sin x$
10. $g(\theta)=\dfrac{\cos\theta}{\theta}$
11. $h(\theta)=\dfrac{\sin\theta}{\theta}$
12. $\dfrac{d}{dx}\left[\dfrac{x-3}{2x+1}\right]$
13. $\dfrac{d}{dt}\left[\dfrac{t^2+2}{2t-7}\right]$
14. $\dfrac{d}{dx}\left[\dfrac{2x^2-3x+1}{x}\right]$
15. $\dfrac{d}{dx}\left[2x^3+5x^2+\dfrac{2}{x}\right]$
16. $\dfrac{d}{dx}[-\cos x+x\sin x]$
17. $f(x)=\dfrac{\sin x}{x^2}$
18. Find an equation of the tangent line to $f(x)=\dfrac{\sin x}{x}$ at the point $(\pi,0)$