

Initials: _____

2.4 Day 5 – Level 1

Find the derivative of $f(x) = \frac{x}{\sqrt{2x-1}}$. $= \frac{x}{(2x-1)^{1/2}} = x(2x-1)^{-1/2}$

$$f'(x) = \frac{(1)(2x-1)^{1/2} - x\left(\frac{1}{2}(2x-1)^{-1/2} \cdot 2\right)}{\left((2x-1)^{1/2}\right)^2}$$

$$= \frac{(2x-1)^{1/2} - x(2x-1)^{-1/2}}{2x-1}$$

GCF:
 $(2x-1)^{-1/2}$

$$= \frac{(2x-1)^{-1/2}(2x-1-x)}{(2x-1)^1}$$

$$= \boxed{\frac{x-1}{(2x-1)^{3/2}}}$$

OR

$$f'(x) = (1)(2x-1)^{-1/2} + x\left(-\frac{1}{2}(2x-1)^{-3/2} \cdot 2\right)$$
$$= (2x-1)^{-1/2} - x(2x-1)^{-3/2}$$

GCF:
 $(2x-1)^{-3/2}$

$$= (2x-1)^{-3/2}(2x-1-x)$$

$$= \boxed{\frac{x-1}{(2x-1)^{3/2}}}$$

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2.4 Day 5 – Level 2

Find the derivative of $g(x) = \left(\frac{1-2x}{1+x}\right)^3$.

$$g'(x) = 3\left(\frac{1-2x}{1+x}\right)^2 \cdot \frac{-2(1+x) - (1)(1-2x)}{(1+x)^2}$$

$$= \frac{3(1-2x)^2}{(1+x)^2} \cdot \frac{-3}{(1+x)^2}$$

$$= \boxed{\frac{-9(1-2x)^2}{(1+x)^4}}$$

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2.4 Day 5 – Level 3

Write the equation of the tangent line to $y = \cos^2(3x)$ at the point where $x = \frac{\pi}{12}$.

$$\begin{aligned}y &= (\cos(3x))^2 \\y' &= 2 \cos(3x) \cdot -\sin(3x) \cdot 3 \\y' &= -6 \cos(3x) \sin(3x) \\y' \left(\frac{\pi}{12} \right) &= -6 \cos \left(3 \cdot \frac{\pi}{12} \right) \sin \left(\frac{3\pi}{12} \right) \\&= -6 \cos \left(\frac{\pi}{4} \right) \sin \left(\frac{\pi}{4} \right) \\&= -6 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \\&= -6 \left(\frac{1}{2} \right) = -3\end{aligned}$$

$$\begin{aligned}y \left(\frac{\pi}{12} \right) &= \cos^2 \left(\frac{3\pi}{12} \right) = \\&= \left(\cos \left(\frac{\pi}{4} \right) \right)^2 \\&= \left(\frac{\sqrt{2}}{2} \right)^2 = \frac{1}{2}\end{aligned}$$

$$\left(\frac{\pi}{12}, \frac{1}{2} \right), m = -3$$

$$y - \frac{1}{2} = -3 \left(x - \frac{\pi}{12} \right)$$

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2.4 Day 5 – Level 4

Write the equation of the tangent line to $f(x) = \sqrt{x}(2-x)^2$ at the point (4, 8).

$$\begin{aligned}f(x) &= x^{1/2} \cdot (2-x)^2 \\f'(x) &= \frac{1}{2} x^{-1/2} (2-x)^2 + x^{1/2} \cdot 2(2-x) \cdot -1 \\&\quad \text{OR} \\&= \frac{1}{2} x^{-1/2} (2-x)^2 - 2x^{1/2} (2-x) \\&\quad \text{GCF: } x^{-1/2} (2-x) \\&= x^{-1/2} (2-x) \left(\frac{1}{2} (2-x) - \frac{2x \cdot 2}{1 \cdot 2} \right) \\&= \frac{2-x}{x^{1/2}} \cdot \frac{2-x-4x}{2} \\&= \frac{(2-x)(2-5x)}{2\sqrt{x}}\end{aligned}$$
$$\begin{aligned}&= \frac{(2-x)^2}{2\sqrt{x}} - \frac{2\sqrt{x}(2-x) \cdot 2\sqrt{x}}{1 \cdot 2\sqrt{x}} \\&= \frac{(2-x)^2 - 4x(2-x)}{2\sqrt{x}} \\&\quad \text{GCF: } (2-x) \\&= \frac{(2-x)(2-x-4x)}{2\sqrt{x}} \\&= \frac{(2-x)(2-5x)}{2\sqrt{x}}\end{aligned}$$

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2.4 Day 5 – Level 5

Find the second derivative of $f(x) = \sin x^2$. Then evaluate $f''(x)$ at $(0,1)$.

$$\begin{aligned} f'(x) &= \cos(x^2) \cdot 2x \\ &= 2x \cdot \cos(x^2) \end{aligned}$$

$$\begin{aligned} f''(x) &= 2 \cos(x^2) + 2x \cdot -\sin(x^2) \cdot 2x \\ &= 2 \cos(x^2) - 4x^2 \sin(x^2) \end{aligned}$$

$$\begin{aligned} f''(0) &= 2 \cos(0) - 4(0)^2 \sin(0) \\ &= \boxed{2} \end{aligned}$$

Initials: _____

2.4 Day 5 – Level 6

a) At what point(s) does the function $y = \sqrt[3]{(x^2-1)^2}$ have a horizontal tangent line?

$$\begin{aligned} y &= (x^2-1)^{2/3} \\ y' &= \frac{2}{3} (x^2-1)^{-1/3} \cdot 2x \\ y' &= \frac{4x}{3(x^2-1)^{1/3}} = 0 \end{aligned}$$

$$m=0$$

$$4x=0$$

$$x=0$$

$$y = (-1)^{2/3} = 1$$

$$\boxed{(0,1)}$$

b) At what point(s) is the derivative undefined? m undef.

$$\text{for } y' = \frac{4x}{3(x^2-1)^{1/3}} \text{ to be undefined, } 3(x^2-1)^{1/3} = 0$$

$$x^2-1=0$$

$$x = \pm 1 = 0$$

$$y = ((\pm 1)^2 - 1)^{2/3} = 0$$

$$\boxed{(1,0), (-1,0)}$$