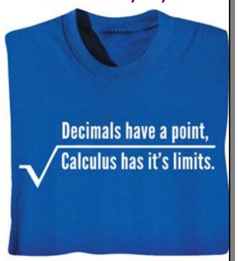
2.4A - The Chain Rule 10/9/18



Homework:
- Section 2.4A
- Formula Quiz Thursday!

Objective: Find the derivative of a function using the Chain Rule

Do Now: $6(3x-2)$

Find the following derivatives.

$f'(x) = 2(3x-2)\cdot 3$

1. $f(x) = (3x-2)^2$

$f(x) = 9x^2 - 12x + 4$

$f'(x) = 18x - 12$

2. $f(x) = (2x+3)^3$ 3.

$f(x) = (2x+3)(4x^2+12x+9)$

$f(x) = 8x^3 + 36x^2 + 54x + 27$

$f'(x) = 24x^2 + 72x + 54$

$f(x) = \dfrac{1}{(2x-3)^2}$

$= \dfrac{1}{4x^2-12x+9}$

$f'(x) = \dfrac{-4}{(2x-3)^3}$

$= \dfrac{-8x+12}{(2x-3)^4} = \dfrac{-4\cancel{(2x-3)}}{(2x-3)^{\cancel{4}\,3}}$

Find $s'(x)$ for $s(x) = \sqrt{7x^2+5x}$.

$S(x) = (7x^2+5x)^{1/2}$

$S'(x) = \frac{1}{2}(7x^2+5x)^{-1/2} \cdot (14x+5)$

$= \dfrac{14x+5}{2\sqrt{7x^2+5x}}$

The Chain Rule is used when you are given composite functions $f(g(x))$.

Without Chain Rule

$y = x^2 + 1$ $\quad y = \sin x$

$y = 3x + 2$ $\quad y = \tan x$

With Chain Rule

$y = \sqrt{x^2 + 1}$ $\quad y = \sin(6x)$

$y = (3x+2)^2$ $\quad y = \tan(x^2)$

$y = \tan^2 x$

$= (\tan x)^2$

THEOREM 2.10 The Chain Rule

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

Find $f'(x)$ and $f''(x)$.

$$f(x) = (2x+5)^{10}$$

$f'(x) = 10(2x+5)^9 \cdot 2$

$= 20(2x+5)^9$

$f''(x) = 180(2x+5)^8 \cdot 2$

$f''(x) = 360(2x+5)^8$

Find $f'(x)$.

$$f(x)=\sqrt{(2x+5)^3} = (2x+5)^{3/2}$$

$$f'(x) = \frac{3}{2}(2x+5)^{1/2} \cdot 2$$

$$= 3\sqrt{2x+5}$$

Find $f'(t)$.

$$f(t)=\sin^3\left(3t^2\right) = \left(\sin\left(3t^2\right)\right)^3$$

$$f'(t) = 3\left(\sin(3t^2)\right)^2 \cdot \cos(3t^2) \cdot 6t$$

$$f'(t) = 18t\,\sin^2(3t^2)\cos(3t^2)$$

$$= \underbrace{2\sin(3t^2)\cos(3t^2)} \cdot 9t\,\sin(3t^2)$$

$$= \sin(6t^2) \cdot 9t\,\sin(3t^2)$$

$$f'(t)=18t\sin^2\left(3t^2\right)\cos\left(3t^2\right)$$

$$f'(t)=9t\sin\left(3t^2\right)\sin\left(6t^2\right)$$

Find g'.

$$g(x)=\underbrace{(2x+1)^3}_{f}\cdot\underbrace{(x^2+x)}_{g}$$

$$g'(x)=3(2x+1)^2\cdot 2(x^2+x)+(2x+1)(2x+1)^3$$

$$g'(x)=6(2x+1)^2(x^2+x)+(2x+1)^4$$

$$g'(x)=(2x+1)^2\left(\overset{6(x^2+x)}{6x^2+6x}+\overset{(2x+1)^2}{4x^2+4x+1}\right)$$

$$\boxed{g'(x)=(2x+1)^2(10x^2+10x+1)}$$

$$g'(x)=(2x+1)^2(10x^2+10x+1)$$

Find y'.

$$y=\sqrt{x}(1-2x)^2=x^{1/2}(1-2x)^2$$

$$y'=\frac{1}{2}x^{-1/2}(1-2x)^2+x^{1/2}\cdot 2(1-2x)(-2)$$

$$y'=\frac{(1-2x)^2}{2\sqrt{x}}-4\sqrt{x}(1-2x)$$

$$y'=(1-2x)\left(\frac{1-2x}{2\sqrt{x}}-\frac{4\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}}\right)$$

$$y'=(1-2x)\left(\frac{1-2x-8x}{2\sqrt{x}}\right)=\frac{(1-2x)(1-10x)}{2\sqrt{x}}$$

$$y'=\frac{(1-2x)(1-10x)}{2\sqrt{x}}$$

Find y'.

$$y = \frac{-5}{(x^2-5x-6)^2} = -5(x^2-5x-6)^{-2}$$

$$y' = 10(x^2-5x-6)^{-3}(2x-5)$$

$$= \frac{10(2x-5)}{(x^2-5x-6)^3}$$

Do we need the quotient rule here?