x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	8	-3	-5
6	3	-2	4	5
8	-1	3	π	4
1	2	-6	5	0

Given the table above, find the value of the derivative of each function at $x = 3$.

1. $\frac{d}{dx}(f(x)+g(x)) = f'(x) + g'(x)$

$f'(3) + g'(3) = -3 + -5 = \boxed{-8}$

2. $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$= f'(3)g(3) + f(3)g'(3)$

$= -3(8) + 1(-5) = -24 + -5 = \boxed{-29}$

3. $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right) = \frac{g'(x)f(x) - f'(x)g(x)}{f^2(x)}$

$= \frac{-5(1) - (-3)(8)}{1^2} = -5 + 24 = \boxed{19}$

4. $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

$f'(g(3)) \cdot g'(3) = f'(8) \cdot -5$

$= \pi(-5) = \boxed{-5\pi}$

5. $\frac{d}{dx}\left(\frac{1}{\sqrt[3]{g(x)}}\right) = \frac{d}{dx}(g(x)^{-1/3}) = -\frac{1}{3}(g(x))^{-4/3} \cdot g'(x)$

$= \frac{-g'(3)}{3g(3)^{4/3}} = \frac{-(-5)}{3(8)^{4/3}} = \frac{5}{3(16)} = \boxed{\frac{5}{48}}$

6. $\frac{d}{dx}(f^3(x)g(x)) = 3f^2(x)f'(x)g(x) + f^3(x)g'(x)$

$= 3f^2(3)f'(3)g(3) + f^3(3)g'(3)$

$= 3(1)^2(-3)(8) + 1^3(-5)$

$= -72 - 5 = \boxed{-77}$

7. The table below gives some values of the derivative of some function f. Complete the table by finding, if possible, the derivative of each of the following functions.

$g(x) = f(x) - 2$ $\quad g'(x) = f'(x)$

$h(x) = 2f(x)$ $\quad h'(x) = 2f'(x)$

$r(x) = f(-3x)$ $\quad r'(x) = f'(-3x) \cdot -3$

$s(x) = f(2x+1)$ $\quad s'(x) = f'(2x+1) \cdot 2$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$	$-3f'(6)$ NOT Possible	$-3f'(3)$ $-3(-4)=12$	$-3f'(0)$ $-3(-\frac{1}{3})=1$	$-3f'(-3)$ NOT Possible	$-3f'(-6)$ NOT Possible	$-3f'(-9)$ NOT Possible
$s'(x)$	$2f'(-3)$ NOT Possible	$2f'(-1)$ $2(\frac{2}{3})=\frac{4}{3}$	$2f'(1)$ $2(-1)=-2$	$2f'(3)$ $2(-4)=-8$	$2f'(5)$ NOT Possible	$2f'(7)$ NOT Possible

Calculate the derivative for each of the following functions. Show all work in your notebook.

$f(x) = \cos x^{1/4} + (\cos x)^{1/4}$

8. $f(x) = \sqrt[5]{\dfrac{2x-3}{4x^2+1}}$

$f(x) = \left(\dfrac{2x-3}{4x^2+1}\right)^{\frac{1}{5}}$

$f'(x) = \frac{1}{5}\left(\frac{2x-3}{4x^2+1}\right)^{\frac{-4}{5}}\left(\frac{2(4x^2+1) - 8x(2x-3)}{(4x^2+1)^2}\right)$

$f'(x) = \frac{1}{5}\dfrac{(4x^2+1)^{4/5}}{(2x-3)^{4/5}}\left(\dfrac{8x^2+2-16x^2+24x}{(4x^2+1)^{2\,6/5}}\right)$

$f'(x) = \dfrac{-8x^2+24x+2}{5(2x-3)^{4/5}(4x^2+1)^{6/5}}$

$f'(x) = \dfrac{-2}{5}\dfrac{(4x^2-12x-1)}{(2x-3)^{4/5}(4x^2+1)^{6/5}}$

9. $f(x) = \cos\sqrt[4]{x} + \sqrt[4]{\cos x}$

$f'(x) = -\sin\sqrt[4]{x}\left(\frac{1}{4}x^{-3/4}\right) + \frac{1}{4}(\cos x)^{-3/4}(-\sin x)$

$f'(x) = \dfrac{-\sin\sqrt[4]{x}}{4x^{3/4}} - \dfrac{\sin x}{4(\cos x)^{3/4}}$

10. $f(x) = \sin^2\left(\dfrac{3x-17}{3x+17}\right) + \cos^2\left(\dfrac{3x-17}{3x+17}\right)$

$f(x) = 1$

$f'(x) = 0$

11. $f(x) = \cos(\cot(x^2))$

$f'(x) = -\sin(\cot x^2)\cdot(-\csc^2 x^2)\cdot 2x$

$f'(x) = 2x\sin(\cot(x^2))\csc^2(x^2)$

12. $f(x) = \sin\sqrt{x}$

$f(x) = \sin(x)^{1/2}$

$f'(x) = \cos(x)^{1/2}\cdot\frac{1}{2}x^{-1/2}$

$f'(x) = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

13. $f(x) = 2x^2\sec(x-2)$

$f'(x) = 4x\sec(x-2) + 2x^2\sec(x-2)\tan(x-2)$

$f'(x) = 2x\sec(x-2)(2 + x\tan(x-2))$

Choose all of the following that is a derivative of the given function.

14. $f(x) = 2\cos x + 5\sin^2(\pi x)$

$f'(x) = -2\sin x + 10\sin(\pi x)\cos(\pi x)(\pi)$

$f'(x) = -2\sin x + 10\pi\sin(\pi x)\cos(\pi x)$

$= -2\sin x + 5\pi\sin(2\pi x)$

a. $f'(x) = -2\sin x + 10\pi\sin(\pi x)\cos(\pi x)$

b. $f'(x) = -2\sin x - 10\pi\sin(\pi x)\cos(\pi x)$

c. $f'(x) = -2\sin x + 5\pi\sin(2\pi x)$

d. $f'(x) = -2\sin x - 5\pi\sin(2\pi x)$

15. $f(x)=\sin(3x-6x^2)\cos(3x-6x^2)$ $f(x)=\frac{1}{2}\sin(6x-12x^2)$

$f'(x)=\frac{1}{2}\cos(6x-12x^2)(6-24x)$

$=(3-12x)\cos(6x-12x^2)=(3-12x)(\cos^2(3x-6x^2)-\sin^2(3x-6x^2))$

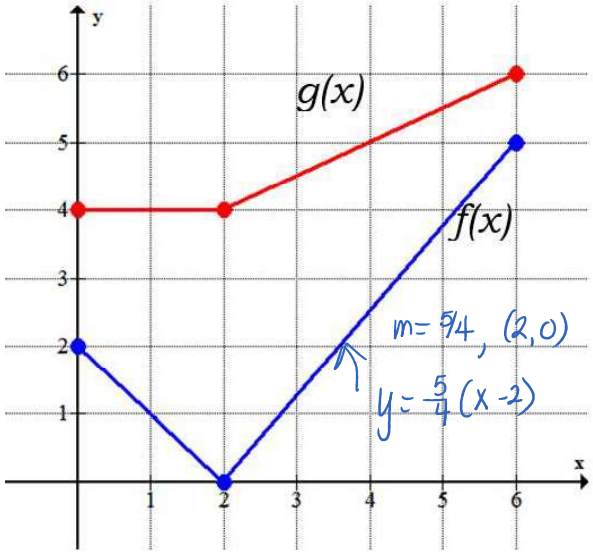
a. $f'(x)=3-12x$

b. $f'(x)=(3-12x)(\cos^2(3x-6x^2)-\sin^2(3x-6x^2))$

c. $f'(x)=(3-12x)\cos(6x-12x^2)$

d. $f'(x)=(3-12x)(\cos^2(3x-6x^2)+\sin^2(3x-6x^2))$

16. Let $r(x)=f(g(x))$ and $s(x)=g(f(x))$ where f and g are shown in the figure below.



a. Find $r'(1)$. $r'(1)=f'(g(1))\cdot g'(1)$

$r'(1)=f'(4)\cdot 0$

$=\frac{5}{4}\cdot 0=\boxed{0}$

b. Find $s'(4)$.

$f(4)=\frac{5}{4}(4-2)=\frac{5}{2}$

$s'(4)=g'(f(4))\cdot f'(4)$

$=g'(\frac{5}{2})\cdot\frac{5}{4}$

$=\frac{1}{2}\cdot\frac{5}{4}=\boxed{\frac{5}{8}}$