

1. Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ at $(5, -3)$.

$$2x - 2yy' = 0$$

$$y' = \frac{x}{y}$$

$$y'|_{(5,-3)} = \frac{5}{-3}$$

2. Find $\frac{dy}{dx}$ given $xy + y = 8$.

$$xy' + y + y' = 0$$

$$y'(x+1) = -y$$

$$y' = \frac{-y}{x+1}$$

3. Find $\frac{dy}{dx}$ for $\sin(xy) = 1$.

$$\cos(xy)(y + xy') = 0$$

$$y \cos(xy) + xy' \cos(xy) = 0$$

$$y' = \frac{-y \cos(xy)}{x \cos(xy)}$$

$$y' = \frac{-y}{x}$$

4. Find $\frac{d^2y}{dx^2}$ given $x^2 + y^2 = 100$.

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

$$y'' = \frac{-y + xy'}{y^2}$$

$$y'' = \frac{-y + x(-\frac{x}{y})}{y^2}$$

$$y'' = \frac{-y^2 - x^2}{y^3} = \frac{-100}{y^3}$$

5. Find $\frac{dy}{dx}$ given $x^2y + xy^2 = 2x$.

$$2xy + x^2y' + y^2 + 2xyy' = 2$$

$$y'(x^2 + 2xy) = 2 - 2xy - y^2$$

$$y' = \frac{2 - 2xy - y^2}{x^2 + 2xy}$$

6. Find $\frac{dy}{dx}$ for $(x+y)^2 + y = 2$ at $(0, 1)$.

$$2(x+y)(1+y') + y' = 0$$

$$(2x+2y)(1+y') + y' = 0$$

$$2x + 2xy' + 2y + 2yy' + y' = 0$$

$$y' = \frac{-2x-2y}{2x+2y+1}$$

$$y'|_{(0,1)} = \frac{-2}{3}$$

7. Find the points where the curve $x^2 + 4y^2 - 4x + 16y + 4 = 0$ has horizontal and vertical tangent lines.

Horizontal:	Vertical:
$(2, 0)$	$(6, -2)$
$(2, -4)$	$(-2, -2)$

$$2x + 8yy' - 4 + 16y' = 0$$

$$y' = \frac{4-2x}{8y+16} = \frac{2-x}{4y+8}$$

Horizontal: $2-x=0$
 $x=2$

$$4+4y^2-8+16y+4=0$$

$$4y^2+16y=0$$

$$4y(y+4)=0$$

$$y=0, -4$$

Vertical: $4y+8=0$
 $y=-2$

$$x^2-4x-12=0$$

$$(x-6)(x+2)=0$$

$$x=6, -2$$

8. Find the equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} - 1 = y$ at $(9, 4)$.

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y' = y'$$

$$y'(\frac{1}{2\sqrt{y}} - 1) = \frac{-1}{2\sqrt{x}}$$

$$y' = \frac{-1}{2\sqrt{x}(\frac{1}{2\sqrt{y}} - 1)}$$

$$y'|_{(9,4)} = \frac{-1}{6(\frac{1}{4}-1)}$$

$$= \frac{-1}{-\frac{9}{2}}$$

$$= \frac{2}{9}$$

$$y-4 = \frac{2}{9}(x-9)$$