1. Find the equation of the tangent and normal line to the curve $x^3 + y^3 = 2xy$ at $(1,1)$.

$y'|_{(1,1)} = \frac{2-3}{3-2} = -1$ $\boxed{y-1=-1(x-1) \quad y-1=1(x-1)}$

$3x^2+3y^2y' = 2y+2xy'$

$y'(3y^2-2x) = 2y-3x^2$

$y' = \frac{2y-3x^2}{3y^2-2x}$

2. <u>AP Question (Calculator Allowed)</u>:

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$

a. Find $\frac{dy}{dx}$.

$6y^2y' + 12xy + 6x^2y' - 24x + 6y' = 0$

$y'(6y^2+6x^2+6) = 24x-12xy$

$y' = \frac{24x-12xy}{6y^2+6x^2+6} = \boxed{\frac{4x-2xy}{y^2+x^2+1}}$

b. Write an equation of each horizontal tangent line to the curve.

$4x-2xy=0$

$2x(2-y)=0$

$x=0 \quad y=2$

$x=0 \rightarrow 2y^3+6y-1=0$

$\boxed{y=.165}$

$y=2 \rightarrow 16+12x^2-12x^2+12=1$

$28 \neq 1$

There are NO points on the curve where $y=2$.

c. The line through the origin with slope -1 is tangent to the curve at point *P*.

* on the line + curve *

Find the *x* and *y* coordinates of point *P*. $\underbrace{(0,0)\ m=-1}_{y=-x}$

$\frac{4x-2xy}{x^2+y^2+1} = -1$

$\frac{4x-2x(-x)}{x^2+(-x)^2+1} = -1$

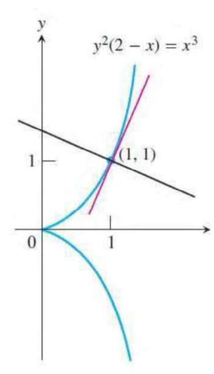
$4x+2x^2 = -2x^2-1$

$4x^2+4x+1=0$

$(2x+1)^2=0 \Rightarrow x=-\frac{1}{2}$

$\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$

3. The Cissoid of Diocles (pictured on right) dates from about 200 B.C.



a. Find equations for the tangent and normal to the Cissoid of Diocles, $y^2(2-x) = x^3$, at the point $(1,1)$.

$2yy'(2-x) + y^2(-1) = 3x^2$

$2yy' = \frac{3x^2+y^2}{2-x}$

$y' = \frac{3x^2+y^2}{2y(2-x)}$

$y'|_{(1,1)} = \frac{3+1}{2(2-1)} = \frac{4}{2} = 2$

$\boxed{\begin{array}{l}\text{Tangent: } y-1=2(x-1) \\ \text{normal: } y-1=-\frac{1}{2}(x-1)\end{array}}$

b. Explain how to reproduce the graph on a graphing calculator.

Solve for y and enter into $y_1 + y_2$.

$y^2 = \frac{x^3}{2-x}$

$y = \pm\sqrt{\frac{x^3}{2-x}}$

4. Find the points where the curve $25x^2 + 16y^2 + 200x - 160y + 400 = 0$ has horizontal and vertical tangent lines.

$50x + 32yy' + 200 - 160y' = 0$

$y'(32y-160) = -50x-200$

$y' = \frac{-50x-200}{32y-160} = \frac{100+25x}{80-16y}$

Horizontal: $100+25x=0$

$25x=-100$

$x=-4$

$16y^2-160y=0$

$16y(y-10)$

$y=0 \quad y=10$

$\boxed{\text{Horizontal: } (-4,0)\ (-4,10)}$

Vertical: $80-16y=0$

$y=5$

$25x^2+200x=0$

$x^2+8x=0$

$x(x+8)=0$

$x=0,\ x=-8$

$\boxed{\text{Vertical: } (0,5)\ (-8,5)}$

5. <u>AP Question (Calculator Allowed)</u>:

Consider the curve $xy^2 - 3y - 6x^2 = 3$

a. Show that $\frac{dy}{dx} = \frac{12x - y^2}{2xy - 3}$

$1y^2 + x2yy' - 3y' - 12x = 0$

$y'(2xy-3) = 12x - y^2$

$\boxed{y' = \frac{12x-y^2}{2xy-3}}$

b. Find the coordinates where the curve has a horizontal tangent line.

$y'=0 \Rightarrow 12x-y^2=0$

$12x=y^2$

$x=\frac{y^2}{12}$

$\frac{y^2}{12}\cdot y^2 - 3y - 6\left(\frac{y^4}{144}\right) = 3$

$\frac{y^4}{12} - 3y - \frac{y^4}{24} = 3$

$2y^4 - 72y - y^4 = 72$

$y^4 - 72y - 72 = 0$

$y=-.9868 \quad y=4.4509$

$x=\frac{y^2}{12}$

$\boxed{(.0811, -.9868) \quad (1.6509, 4.4509)}$

c. Find the coordinates where the curve has a vertical tangent line.

$y'=\text{und.} \Rightarrow 2xy-3=0$

$y=\frac{3}{2x}$

$x\left(\frac{3}{2x}\right)^2 - 3\left(\frac{3}{2x}\right) - 6x^2 = 3$

$\frac{9x}{4x^2} - \frac{9}{2x} - 6x^2 = 3$

$\frac{9}{4x} - \frac{9}{2x} - 6x^2 = 3$

$9 - 18 - 24x^3 = 12x$

$-24x^3 - 12x - 9 = 0 \Rightarrow x = -\frac{1}{2} \rightarrow y = -3$

$\boxed{\left(-\frac{1}{2}, -3\right)}$

6. Find $\frac{dy}{dx}$ for $x = \sec\left(\frac{1}{y}\right)$.

$1 = \sec\left(\frac{1}{y}\right)\tan\left(\frac{1}{y}\right)\left(\frac{-1}{y^2}\right)y'$

$\boxed{\frac{-y^2}{\sec\left(\frac{1}{y}\right)\tan\left(\frac{1}{y}\right)} = y'}$

7. Find $\frac{d^2y}{dx^2}$ for $1 - xy = x - y$.

$-y - xy' = 1 - y'$

$y'(1-x) = 1+y$

$y' = \frac{1+y}{1-x}$

$y'' = \frac{y'(1-x) - (-1)(1+y)}{(1-x)^2}$

$= \frac{\left(\frac{1+y}{1-x}\right)(1-x) + 1 + y}{(1-x)^2} = \frac{2+2y}{(1-x)^2}$

$\boxed{y'' = \frac{2+2y}{(1-x)^2}}$

8. Find $\frac{dy}{dx}$ for $(x+y)^3 = x^3 + y^3$ at $(-1, 1)$.

$3(x+y)^2(1+y') = 3x^2 + 3y^2y'$

$(x+y)^2 + y'(x+y)^2 = x^2 + y^2y'$

$y'\left((x+y)^2 - y^2\right) = x^2 - (x+y)^2$

$y' = \frac{x^2 - (x+y)^2}{(x+y)^2 - y^2}$

$\boxed{y'\Big|_{(-1,1)} = \frac{(-1)^2 - 0}{0 - 1^2} = -1}$

9. Find $\frac{dy}{dx}$ for $x\cos(y) = 1$ at $\left(2, \frac{\pi}{3}\right)$.

$\cos(y) + x(-\sin(y))y' = 0$

$y' = \frac{-\cos(y)}{-x\sin(y)} = \frac{\cos(y)}{x\sin y}$

$y'\Big|_{(2,\frac{\pi}{3})} = \frac{\cos\left(\frac{\pi}{3}\right)}{2\sin\left(\frac{\pi}{3}\right)} = \frac{\frac{1}{2}}{2\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

$\boxed{y'\Big|_{(2,\frac{\pi}{3})} = \frac{\sqrt{3}}{6}}$

10. Find $\frac{dy}{dx}$ for $x^3y - y = x$.

$3x^2y + x^3y' - y' = 1$

$y'(x^3-1) = 1-3x^2y$

$\boxed{y' = \frac{1-3x^2y}{x^3-1}}$

11. Find $\frac{dy}{dx}$ for $\tan(x+y) = y$.

$\sec^2(x+y)(1+y') = y'$

$\sec^2(x+y) + y'(\sec^2(x+y) - y' = 0$

$y' = \frac{-\sec^2(x+y)}{\sec^2(x+y)-1}$

$= \frac{-\sec^2(x+y)}{\tan^2(x+y)} = \boxed{-\csc^2(x+y)}$

12. Find $\frac{dy}{dx}$ for $2\sin(x)\cos(y) = 1$.

$2\cos(x)\cos y + 2\sin(x)(-\sin(y))(y') = 0$

$(-2\sin(x)\sin y)y' = -2\cos x\cos y$

$y' = \frac{-2\cos x\cos y}{-2\sin x\sin y}$

$\boxed{y' = \cot(x)\cot(y)}$

13. Find $\frac{dy}{dx}$ for $y + \sqrt{xy} = 2$ at $(2,2)$.

$y' + \frac{1}{2}(xy)^{-1/2}(1y + xy') = 0$

$y' + \frac{y}{2\sqrt{xy}} + \frac{xy'}{2\sqrt{xy}} = 0$

$y'\left(1 + \frac{x}{2\sqrt{xy}}\right) = \frac{-y}{2\sqrt{xy}}$

$y' = \frac{\frac{-y}{2\sqrt{xy}}}{1 + \frac{x}{2\sqrt{xy}}}$

$y' = \frac{-y}{2\sqrt{xy} + x}$

$\boxed{y'\big|_{(2,2)} = \frac{-1}{3}}$