2.5 Day 2 - Implicit Differentiation 12/7/18

Homework:
- 2.5B
- Quiz 2.5 Thursday, 12/13

Objective:

Use implicit differentiation to find the derivative of a function.

Do Now: Find $\frac{dy}{dx}$ given

1) $\frac{d}{dx}\left(x^2y + y^2x = -2\right)$ 2)

$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx}\cdot x + y^2 = 0$

$\frac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$

$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$

$\frac{d}{dx}\left(\sec\frac{1}{y} = \pi x\right)$ $\quad \sec(y)$

$\sec\left(\frac{1}{y}\right)\tan\left(\frac{1}{y}\right)\left(-y^{-2}\frac{dy}{dx}\right) = \pi$

$-y^2\left(-\frac{1}{y^2}\frac{dy}{dx}\right) = \left(\frac{\pi}{\sec\left(\frac{1}{y}\right)\tan\left(\frac{1}{y}\right)}\right)-y^2$

$\frac{dy}{dx} = \frac{-\pi y^2}{\sec\left(\frac{1}{y}\right)\tan\left(\frac{1}{y}\right)}$

$\frac{dy}{dx} = -\pi y^2\cos\left(\frac{1}{y}\right)\cot\left(\frac{1}{y}\right)$

Homework Questions?

Examples... Find $y' = \frac{dy}{dx}$

1) $\frac{d}{dx}\left(\tan(2x) = x(1+\cot(y))\right)$

$2\sec^2(2x) = 1 + \cot(y) + x\left(-\csc^2(y)\frac{dy}{dx}\right)$

$2\sec^2(2x) - 1 - \cot(y) = -x\csc^2(y)\frac{dy}{dx}$

$$-\frac{2\sec^2(2x) - 1 - \cot(y)}{x\csc^2(y)} = \frac{dy}{dx}$$

Your Turn... Find y'

2) $(2x+2y)^3 = 8x^3 + 8y^3$ *Hint: Cube the binomial first.*

$8x^3 + 24x^2y + 24xy^2 + 8y^3 = 8x^3 + 8y^3$

$\frac{24x^2y + 24xy^2 = 0}{24}$

$\frac{d}{dx}\left(x^2y + xy^2 = 0\right)$

$2xy + x^2\frac{dy}{dx} + y^2 + x\left(2y\frac{dy}{dx}\right) = 0$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

3) Evaluate $\frac{dy}{dx}$ when $x = 6$ given:

$\frac{d}{dx}(xy = -6)$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$\left.\frac{dy}{dx}\right|_{(6,-1)} = -\frac{(-1)}{6} = \boxed{\frac{1}{6}}$

$xy = -6, x = 6$

$6y = -6$

$y = -1$

$(6, -1)$

4) Evaluate $\frac{dy}{dx}$ when $x = 6$ given:

$\frac{d}{dx}\left(y^2 = \frac{x+3}{x-5}\right)$

$2y\frac{dy}{dx} = \frac{1(x-5) - 1(x+3)}{(x-5)^2} \Rightarrow x - 5 - x - 3$

$\frac{1}{2y} \cdot 2y\frac{dy}{dx} = \frac{-8}{(x-5)^2} \cdot \frac{1}{2y}$

$\boxed{\frac{dy}{dx} = \frac{-4}{y(x-5)^2}}$

$y^2 = \frac{x+3}{x-5}, x = 6$

$y^2 = \frac{6+3}{6-5} = 9$

$y^2 = 9$

$y = \pm 3$

$(6,3), (6,-3)$

$(6,3)$

$\left.\frac{dy}{dx}\right|_{(6,3)} = \frac{-4}{3(6-5)^2} = \boxed{\frac{-4}{3}}$

$(6,-3)$

$\left.\frac{dy}{dx}\right|_{(6,-3)} = \frac{-4}{-3(6-5)^2} = \boxed{\frac{4}{3}}$

5) Find the point(s) where there is a horizontal tangent line, then find the points where there is a vertical tangent line.

$\frac{d}{dx}(2x^2 + y^2 + 6y = 16)$

$4x + 2y\frac{dy}{dx} + 6\frac{dy}{dx} = 0$

$2x + y\frac{dy}{dx} + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{y+3}$

Horiz tan line $\Rightarrow$

$\frac{dy}{dx} = \frac{-2x}{y+3} = \frac{0}{1}$

$-2x = 0$

$x = 0$

$y^2 + 6y = 16$

$y^2 + 6y - 16 = 0$

$(y+8)(y-2) = 0$

$y = -8, 2$

$(0, -8)$

$(0, 2)$

$2x^2 + y^2 + 6y = 16$

Vert tan line

$\frac{dy}{dx} = \frac{-2x}{y+3}$ und.

$y + 3 = 0$

$y = -3$

$2x^2 + (-3)^2 + 6(-3) = 16$

$2x^2 + 9 - 18 = 16$

$2x^2 - 9 = 16$

$2x^2 = 25$

$x^2 = \frac{25}{2}$

$x = \pm\frac{5}{\sqrt{2}}$

$\left(\frac{5}{\sqrt{2}}, -3\right)$

$\left(-\frac{5}{\sqrt{2}}, -3\right)$

6) Find the point(s) where there is a horizontal tangent line, then find the points where there is a vertical tangent line.

$$9x^2 + y^2 - 6x = 15$$

More Fun!

Evaluate $\frac{dy}{dx}$ for the given point.

$$x \sin y = 1 \text{ at } (2, \frac{\pi}{6})$$