2.5 Day 3 - Implicit Differentiation 12/10/18

Homework:
- 2.5C
- Quiz 2.5 Thursday, 12/13

Objective:

Use implicit differentiation to find second derivatives and write equations of the tangent and normal line

Do Now: Find $\frac{dy}{dx}$ given

$\frac{d}{dx}\left(\cot(x+y)=y\right)$

$-\csc^2(x+y)\left(1+\frac{dy}{dx}\right)=\frac{dy}{dx}$

$-\csc^2(x+y)-\csc^2(x+y)\frac{dy}{dx}=\frac{dy}{dx}$

$-\csc^2(x+y)=\frac{dy}{dx}+\csc^2(x+y)\frac{dy}{dx}$

$-\csc^2(x+y)=\frac{dy}{dx}\left(1+\csc^2(x+y)\right)$

$\frac{-\csc^2(x+y)}{1+\csc^2(x+y)}=\frac{dy}{dx}$

Homework Questions?

$\frac{d}{dx}\left(\tan(x+y)=x\right)$ $(0,0)$

$\sec^2(x+y)\left(1+\frac{dy}{dx}\right)=1$ $\rightarrow$ $1+\frac{dy}{dx}=\cos^2(x+y)$

$\sec^2(x+y)+\sec^2(x+y)\frac{dy}{dx}=1$ $\frac{dy}{dx}=\cos^2(x+y)-1$

$\sec^2(x+y)\frac{dy}{dx}=1-\sec^2(x+y)$

$\frac{dy}{dx}=\frac{1-\sec^2(x+y)}{\sec^2(x+y)}=$

$\frac{d}{dx}\left(x\cdot\cos y=1\right)$ $(2,\pi/3)$

$\cos y-x\sin y\frac{dy}{dx}=0$

$-x\sin y\frac{dy}{dx}=-\cos y$

$\frac{dy}{dx}=\frac{\cos y}{x\sin y}=\frac{\cot y}{x}$

$\left.\frac{dy}{dx}\right|_{(2,\pi/3)}=\frac{\cos\pi/3}{2\sin\pi/3}$

$=\frac{\frac{1}{2}}{2\left(\frac{\sqrt{3}}{2}\right)}$

$=\frac{1}{2\sqrt{3}}$

Find y''

1) $\frac{d}{dx}(3x^2+4y^2=4)$

$6x + 8y\frac{dy}{dx} = 0$ $\quad 8y\frac{dy}{dx} = -6x$

$\frac{dy}{dx} = \frac{-6x}{8y} = \frac{-3x}{4y}$

$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{-3x}{4y}\right)$

$\frac{d^2y}{dx^2} = \frac{-3(4y) - (-3x)\left(4\frac{dy}{dx}\right)}{(4y)^2}$

$= \frac{-12y + 12x\left(\frac{-3x}{4y}\right)}{16y^2}$

$= \frac{\left(-12y - \frac{9x^2}{y}\right)y}{(16y^2)y}$

$\frac{d^2y}{dx^2} = \frac{-12y^2 - 9x^2}{16y^3}$

$3x^2+4y^2=4$

$= \frac{-3(4y^2+3x^2)}{16y^3} = \frac{-3(4)}{16y^3} = \boxed{\frac{-3}{4y^3}}$

Find y''

2) $\frac{d}{dx}(x^3 - y^3 = 1)$

$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{x^2}{y^2}\right)$

$\frac{d^2y}{dx^2} = \frac{2xy^2 - x^2\left(2y\frac{dy}{dx}\right)}{(y^2)^2}$

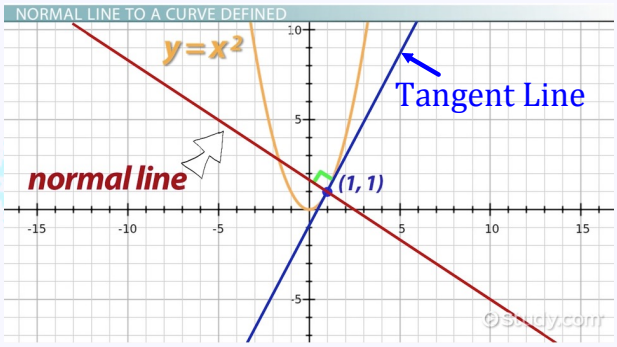
$\frac{d^2y}{dx^2} = \frac{\left(2xy^2 - 2x^2y\left(\frac{x^2}{y^2}\right)\right)y}{y^4 \cdot y}$

$x^3 - y^3 = 1$

$= \frac{2xy^3 - 2x^4}{y^5} = \frac{-2x(-y^3+x^3)}{y^5}$

$= \boxed{\frac{-2x}{y^5}}$

What is a normal line?



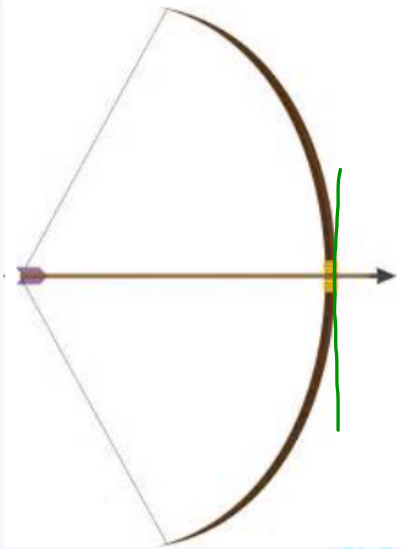
The normal line is ***perpendicular*** to the tangent line.

What does that tell us about the relationship between the slope of the normal line and the slope of the tangent line?

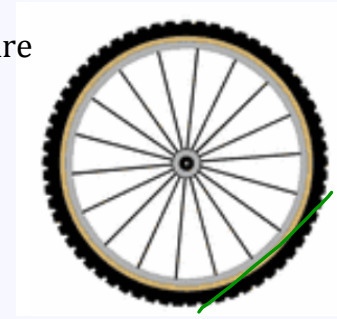
tan line (x,y) $m=\frac{dy}{dx}$

norm line $m_{\perp}$

The bow is the curve and the arrow is the **normal line** at a point on the curve.



The spokes of the wheel are **normal** to the rim.



When you are going fast around a circular track in a car, the force that you feel pushing you outwards is **normal** to the curve of the road.

3) Find the equation of the **tangent line** and the **normal line** to graph at the given point.

$\frac{d}{dx}(x^2 + y^2 = 20)$ $(4,-2)$

$\frac{dy}{dx} = \frac{-x}{y}$ $m = \left.\frac{dy}{dx}\right|_{(4,-2)} = \frac{-4}{-2} = 2$

Eqn of tan line: $y+2 = 2(x-4)$

Eqn of norm line: $y+2 = -\frac{1}{2}(x-4)$

$m_1 = -\frac{1}{2}$

4) Write down the equation of the **tangent line** and the **normal line** to graph at point where $x = 1$.

$(2x^2 - y^3 = -62)$

$2(1)^2 - y^3 = -62$

$2 - y^3 = -62$

$y^3 = 64$

$y = 4$

$\frac{dy}{dx} = \frac{4x}{3y^2}$

$\left.\frac{dy}{dx}\right|_{(1,4)} = \frac{4(1)}{3(4)^2} = \frac{1}{12}$

tan line: $y-4 = \frac{1}{12}(x-1)$

norm line: $y-4 = -12(x-1)$

Exit Card