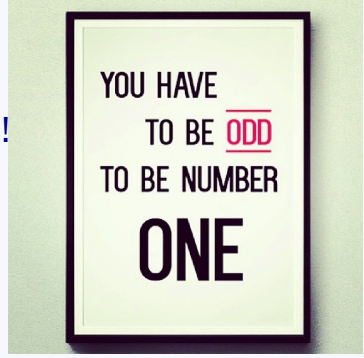
2.5 Day 4 - Implicit Differentiation

12/11/18

Homework:

- 2.5, #2, 4, 54
- Finish Classwork - check answers online!
- Quiz 2.5 Thursday, 12/13



Objective:

Use implicit differentiation to find derivatives and second derivatives

Do Now:

1. Find the slope of the tangent line at (2,-3):

$$2x^3 - x^2y + y^3 - 1 = 0$$

2. Find $\frac{d^2y}{dx^2}$ given: $5x^2 - 2y^2 = 4$

Let's revisit this example from yesterday...

1) $3x^2 + 4y^2 = 4$

$\frac{dy}{dx} = \frac{-3x}{4y}$

$\frac{d^2y}{dx^2} = \frac{-12y^2 - 9x^2}{16y^3}$

Let's revisit this example from yesterday...

2) $x^3 - y^3 = 1$

$$\frac{dy}{dx} = \frac{x^2}{y^2}$$

$$\frac{d^2x}{dy^2} = \frac{2xy^3 - 2x^4}{y^5}$$

Homework Questions?

$(x^2+y^2)^2 = 4x^2y \qquad (1,1)$

$\frac{d}{dx}(x^4+2x^2y^2+y^4 = 4x^2y)$

$4x^3+4xy^2+4x^2y\frac{dy}{dx}+4y^3\frac{dy}{dx} = 8xy+4x^2\frac{dy}{dx}$

$x^3+xy^2+x^2y\frac{dy}{dx}+y^3\frac{dy}{dx} = 2xy+x^2\frac{dy}{dx}$

$\frac{dy}{dx}(x^2y+y^3-x^2) = 2xy-x^3-xy^2$

$\frac{dy}{dx} = \frac{2xy-x^3-xy^2}{x^2y+y^3-x^2}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2-1-1}{1+1-1} = \frac{0}{1} = 0$

$\frac{d}{dx}(25x^2+16y^2+200x-160y+400=0)$

$50x+32y\frac{dy}{dx}+200-160\frac{dy}{dx}=0$

$25x+16y\frac{dy}{dx}+100-80\frac{dy}{dx}=0$

$\frac{dy}{dx}(16y-80) = -25x-100$

$\frac{dy}{dx} = \frac{-25x-100}{16y-80}$

Horiz: $m=0$

$\frac{-25x-100}{16y-80}=0$

$-25x=100$

$x=-4$

$25(16)+16y^2+200(-4)-160y+400=0$

$400+16y^2-800-160y+400=0$

$16y^2-160y=0$

$16y(y-10)=0$

$y=0,10$

$(-4,0),(-4,10)$

Vert: mund

$16y-80=0$

$y=5$

$25x^2+16(25)+200x-160(5)+400=0$

$25x^2+400+200x-800+400=0$

$25x^2+200x=0$

$25x(x+8)=0$

$x=0,-8$

$(0,5),(-8,5)$

Complete the half sheet in your groups.

1. Find $\frac{dy}{dx}$ for $(\sin \pi x + \cos \pi y)^2 = 2$
2. Find the value of $\frac{dy}{dx}$ for $(x+y)^3 = x^3 + y^3$ at the point $(-1,1)$
3. Find $\frac{d^2y}{dx^2}$ for $6x^2 - 7y^3 = 10$

For #4 - #6, find the equations of the tangent and normal lines to the given curves at the given points.

4. $y^2 - 2x - 4y - 1 = 0$ at $(-2,1)$
5. $(-4x+5)^2 = y^3$ at $(1,1)$
6. $x^2 + 4y^2 = 4$ at $\left(\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

$\frac{d}{dx}(\tan(x^2+y) = x)$

$\sec^2(x^2+y)\left(2x + \frac{dy}{dx}\right) = 1$

$2x + \frac{dy}{dx} = \frac{1}{\sec^2(x^2+y)}$

$2x + \frac{dy}{dx} = \cos^2(x^2+y)$

$\frac{dy}{dx} = \cos^2(x^2+y) - 2x$

$2x\sec^2(x^2+y) + \sec^2(x^2+y)\frac{dy}{dx} = 1$

$\sec^2(x^2+y)\frac{dy}{dx} = 1 - 2x\sec^2(x^2+y)$

$\frac{dy}{dx} = \frac{1 - 2x\sec^2(x^2+y)}{\sec^2(x^2+y)}$

Answers to Half Sheet

1) $y' = \dfrac{\cos \pi x}{\sin \pi y}$

2) $y' = -1$

3) $y'' = \dfrac{4(7y^3 - 8x^2)}{49y^5}$

Answers to Half Sheet

4) Tangent Line

$$y-1=-(x+2)$$

Normal Line

$$y-1=(x+2)$$

5) Tangent Line

$$y-1=-\frac{8}{3}(x+2)$$

Normal Line

$$y-1=\frac{3}{8}(x+2)$$

Answers to Half Sheet

6) Tangent Line

$$y+\frac{1}{\sqrt{2}}=\frac{1}{2}(x-\sqrt{2})$$

Normal Line

$$y+\frac{1}{\sqrt{2}}=-2(x-\sqrt{2})$$