Name Answer Key **Date**

Calc I H – 2.5 day 4 – Implicit Differentiation Practice **Period**

1. Find $\frac{dy}{dx}$ for $\left((\sin \pi x + \cos \pi y)^2 = 2\right)\frac{d}{dx}$

$2(\sin\pi x + \cos\pi y)\cdot\left(\pi\cos\pi x - \pi\sin\pi y\frac{dy}{dx}\right) = 0$

$\cancel{2}\pi\cos\pi x\,\cancel{(\sin\pi x + \cos\pi y)} = \cancel{2}\pi\sin\pi y\,\cancel{(\sin\pi x + \cos\pi y)}\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{\cos\pi x}{\sin\pi y}}$$

2. Find the value of $\frac{dy}{dx}$ for $(x+y)^3 = x^3 + y^3$ at the point $(-1,1)$

$(x+y)^3 = (x^2+2xy+y^2)(x+y) = x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$

$= x^2 + 3x^2y + 3xy^2 + y^3$

$\cancel{x^2} + 3x^2y + 3xy^2 + \cancel{y^3} = \cancel{x^3} + \cancel{y^3}$

$3x^2y + 3xy^2 = 0$

$\frac{d}{dx}\left(x^2y + xy^2 = 0\right)$

$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 0$

$(x^2+2xy)\frac{dy}{dx} = -(2xy+y^2)$

$$\boxed{\frac{dy}{dx} = -\frac{2xy+y^2}{x^2+2xy}}$$

$\left.\frac{dy}{dx}\right|_{(-1,1)} = -\frac{-2+1}{1-2}$

$= \boxed{-1}$

3. Find $\frac{d^2y}{dx^2}$ for $\left(6x^2 - 7y^3 = 10\right)\frac{d}{dx}$

$12x - 21y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{12x}{21y^2}$

$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{4x}{7y^2}\right)$

$\frac{d^2y}{dx^2} = \frac{4(7y^2) - (14y\frac{dy}{dx})(4x)}{(7y^2)^2}$

$= \frac{\left(28y^2 - 56xy\frac{dy}{dx}\right)\frac{1}{7}}{(49y^4)\frac{1}{7}}$

$= \frac{4y^2 - 8xy\left(\frac{4x}{7y^2}\right)}{7y^4}$

$= \frac{\left(4y^2 - \frac{32x^2}{7y}\right)7y}{(7y^4)7y} = \boxed{\frac{28y^3 - 32x^2}{49y^5}}$

For #4 - #6, find the equations of the tangent and normal lines to the given curves at the given points.

$\frac{d}{dx}$ 4. $\left(y^2-2x-4y-1=0\right)$ at $(-2,1)$

$$2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$$

$$y\frac{dy}{dx}-1-2\frac{dy}{dx}=0$$

$$(y-2)\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{y-2}$$

$$\left.\frac{dy}{dx}\right|_{(-2,1)}=\frac{1}{1-2}=-1$$

Tan Line: $m=-1$

$y-1=-(x+2)$

Norm Line: $m=1$

$y-1=x+2$

$\frac{d}{dx}$ 5. $\left((-4x+5)^2=y^3\right)$ at $(1,1)$

$$2(-4x+5)(-4)=3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{-8(-4x+5)}{3y^2}=\frac{8(4x-5)}{3y^2}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{8(4-5)}{3}=\frac{-8}{3}$$

Tan Line: $m=\frac{-8}{3}$

$y-1=\frac{-8}{3}(x-1)$

Norm Line: $m=\frac{3}{8}$

$y-1=\frac{3}{8}(x-1)$

$\frac{d}{dx}$ 6 $\left(x^2+4y^2=4\right)$ at $\left(\sqrt{2},-\frac{1}{\sqrt{2}}\right)$

$$2x+8y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-x}{4y}$$

$$\left.\frac{dy}{dx}\right|_{(\sqrt{2},-\frac{1}{\sqrt{2}})}=\frac{-\sqrt{2}}{4(-\frac{1}{\sqrt{2}})}=\frac{2}{4}=\frac{1}{2}$$

Tan Line: $m=\frac{1}{2}$

$y+\frac{1}{\sqrt{2}}=\frac{1}{2}(x-\sqrt{2})$

Norm Line: $m=-2$

$y+\frac{1}{\sqrt{2}}=-2(x-\sqrt{2})$