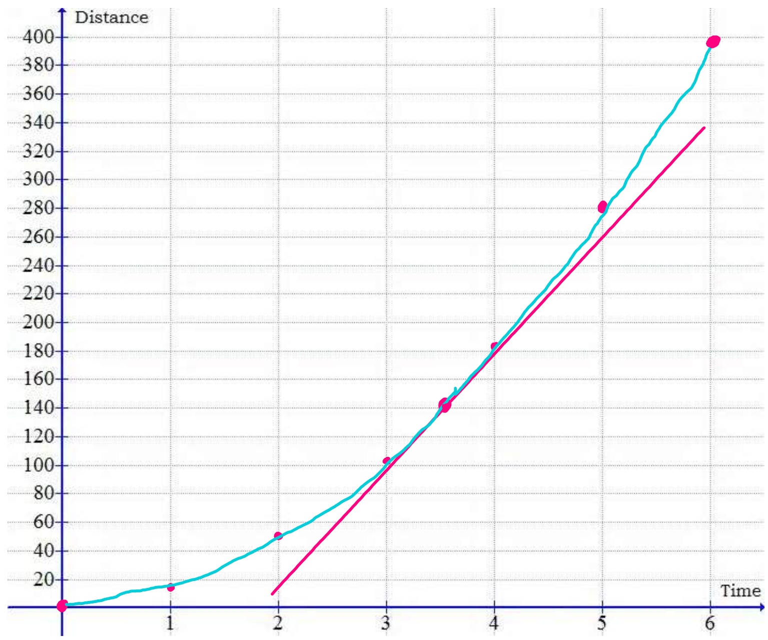
Do Now:

A car company needs more data of a car that they are developing so they decide to perform a test run. Below is a chart representing the distance, in feet, traveled by the car during the first six seconds of its test run. Use the data to provide a sketch.

Time (sec)	Distance (ft)
0	0
1	16
2	52
3	108
4	184
5	280
6	396



1. Using the data above, what is the average velocity of the car during the first two seconds of the test? The last two seconds? Between 3 and 4 seconds?

$\frac{52-0}{2-0} = 26$ ft/sec $\quad \frac{396-184}{6-4} = \frac{212}{2} = 106$ ft/sec $\quad \frac{184-108}{1} = 76$ ft/sec

2. Using the data above, give an estimate of the *instantaneous* velocity of the car at 3.5 seconds.

(5,260)
(2,20)
$\frac{260-20}{5-2} = \frac{240}{3} \approx 80$ ft/sec

3. How does $\underbrace{\text{average velocity}}_{\text{slope of secant line}}$ and $\underbrace{\text{instantaneous velocity}}_{\text{slope of tangent line}}$ relate to your sketch above?

4. Is the car accelerating or decelerating during its test run? Explain graphically.

Since the slopes of the tangent lines are constantly increasing, the velocity is increasing. This means the car is accelerating.

5. The company discovers that the data models the function $s(t) = 10t^2 + 6t$, where s (distance in feet) is a function of t (time in seconds) for $0 \le t \le 6$. Using this fact, can you improve upon your previous estimate for the car's instantaneous velocity at $t = 3.5$?

$s'(t) = v(t) = 20t + 6$ $\quad v(3.5) = 76$ ft/sec

6. Justify the cars acceleration using the velocity function found above.

$s''(t) = v'(t) = a(t) = 20$ $\quad a(3.5) = 20$ ft/sec^2

Since v(t) & a(t) are both positive (same sign/direction), the speed is increasing

Class Work:

1. A dynamite blast propels a heavy rock straight up with a launch velocity of 160ft/sec (about 109 mph). It reaches a height of $s = 160t - 16t^2$ ft after t seconds.

   a. How high does the rock go? $v(t) = 160 - 32t = 0$ $t = 5$ $s(5) = 160(5) - 16(5)^2 = 400$ ft

   b. What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

   $160t - 16t^2 = 256$
   $0 = 16t^2 - 160t + 256$
   $0 = 16(t^2 - 10t + 16)$
   $0 = 16(t-2)(t-8)$

   $t=2$ $v(2) = 96$ ft/sec Speed: 96 ft/sec

   $t=8$ $v(8) = -96$ ft/sec Speed: 96 ft/sec

   c. What is the acceleration of the rock at any time t during its flight (after the blast)?

   $a(t) = -32$ ft/sec^2 → due to gravity

   d. When does the rock hit the ground?

   $160t - 16t^2 = 0$
   $16t(10 - t) = 0$
   $t = 0$ $t = 10$

   10 seconds

2. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

   a. Find the displacement of the particle during the first 2 seconds.

   $s(0) = 3$ $s(2) = 2^2 - 4(2) + 3 = -1$ The particle is displaced 4m left from the starting point.

   b. Find the average velocity of the particle during the first 4 seconds.

   $\frac{s(4) - s(0)}{4 - 0} = \frac{3-3}{4} = 0$ m/sec

   c. Find the instantaneous velocity of the particle when $t = 4$.

   $s'(t) = v(t) = 2t - 4$ $v(4) = 4$ m/sec

   d. Find the acceleration of the particle when $t = 4$.

   $s''(t) = v'(t) = a(t) = 2$ $a(4) = 2$ m/sec^2

   e. Is the speed increasing or decreasing at $t = 4$? Explain your reasoning.

   $\left.\begin{array}{l} a(4) > 0 \\ v(4) > 0 \end{array}\right\}$ Same sign → working together → speed increasing

   f. Describe the motion of the particle. At what values of t does the particle change directions?

   $v(t) = 2t - 4 = 0$
   $t = 2$

   $s(0) = 3$ $s(4) = 3$
   $s(2) = -1$

   Particle moves left when $\underbrace{0 < t < 2}_{v(t)<0}$ & $\underbrace{\text{moves right}}_{v(t)>0}$ when $t > 2$

   g. Where is the particle when $\underbrace{\text{s is a minimum}}_{t=2}$?

   $s(2) = -1$

   (particle is 4 meters to the left of the starting point)

3. <u>Graphing Calculator Active</u>: Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

a. For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

$r'(t) = 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-1)(t-3) = 0$

$v(t)$: $+$ | 1 | $-$ | 3 | $+$

$0 < t < 1$ & $3 < t < 6$ since $r'(t) > 0$

or

$(0,1) \cup (3,6)$

b. For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

$p'(t) = -2\sin\left(\frac{\pi}{4}t\right)\frac{\pi}{4}$

$= -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right) = 0$

$\frac{\pi}{4}t = 0$ $\quad$ $\frac{\pi}{4}t = \pi$

$t = 0$ $\quad$ $t = 4$

$v(t)$: $-$ | 4 | $+$

$0 < t < 1$ & $3 < t < 4$

or

$(0,1) \cup (3,4)$

c. Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down or doing neither at time $t = 3$? Explain your reasoning.

$p'(3) = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right) < 0$

$p''(t) = -\frac{\pi}{2}\cos\left(\frac{\pi}{4}t\right)\cdot\frac{\pi}{4}$

$= -\frac{\pi^2}{8}\cos\left(\frac{\pi}{4}t\right)$

$p''(3) = -\frac{\pi^2}{8}\cos\left(\frac{3\pi}{4}\right) > 0$

Since the values of the velocity & acceleration differ in signs, the particle's speed is slowing down.

d. Find the average rate of change of particle R on the interval $2 \le t \le 4$. What is the instantaneous rate of change of particle R at time $t = 3$?

$r(t) = t^3 - 6t^2 + 9t + 3$ $\quad$ avg $= \frac{r(4) - r(2)}{4 - 2} = \frac{7 - 5}{2} = 1$

$r'(t) = 3t^2 - 12t + 9$

$r'(3) = 3(3)^2 - 12(3) + 9 = 0$

average velocity: 1

instantaneous velocity: 0

e. The position of particle S at time t is given for select values in the table below where $s(t)$ is a differentiable function for $0 \le t \le 6$.

t	0	2	4	6
$s(t)$	0	12	14	-60

Which particle is moving the fastest at time $t = 3$? Show all work to support your answer.
Which particle has the greatest average velocity for $0 \le t \le 6$?

$r'(3) = 0$

$p'(3) = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right) = -\frac{\pi}{2}\left(\frac{\sqrt{2}}{2}\right) = -\frac{\pi\sqrt{2}}{4} \approx -1.111$

$s'(3) = \frac{s(4) - s(2)}{4 - 2} = \frac{14 - 12}{2} = 1$

$|p'(3)|$ is greatest so particle P is moving fastest

$\frac{r(6) - r(0)}{6 - 0} = \frac{57 - 3}{6} = \frac{54}{6} = 9$

$\frac{p(6) - p(0)}{6 - 0} = \frac{0 - 2}{6} = -\frac{1}{3}$

$\frac{s(6) - s(0)}{6 - 0} = \frac{-60 - 0}{6} = -10$

particle R

4. A particle is moving along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = (t-2)^2(t-4)$ where s is measured in meters and t is measured in seconds.

a. Find the instantaneous velocity at any time t.

$$s'(t) = v(t) = 2(t-2)(t-4) + (t-2)^2$$

$$v(t) = (t-2)(2t-8+t-2)$$

$$v(t) = (t-2)(3t-10) = 3t^2-10t-6t+20 = 3t^2-16t+20$$

b. Find the acceleration of the particle at any time t.

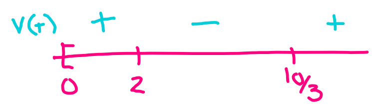
$$a(t) = v'(t) = 6t-16$$

c. Describe the motion of the particle. At what values of t does the particle change direction? On what intervals is it heading forward? On what intervals is it heading backwards?

$$v(t) = 3t^2-16t+20 = 0$$

$$(3t-10)(t-2)$$

$$t = \frac{10}{3} \quad t = 2$$



moving forward on $0<t<2$ & $t>\frac{10}{3}$ since $v(t)>0$

moving backward on $2<t<\frac{10}{3}$ since $v(t)<0$

changes direct at $t=2$ & $t=\frac{10}{3}$

d. Where is the particle when the acceleration is zero?

$$a(t) = 6t-16 = 0$$

$$t = \frac{16}{6} = \frac{8}{3}$$

$$s(8/3) = \left(\frac{8}{3}-2\right)^2\left(\frac{8}{3}-4\right) = \frac{-16}{27}$$

$\frac{16}{27}$ meters to the left of the origin