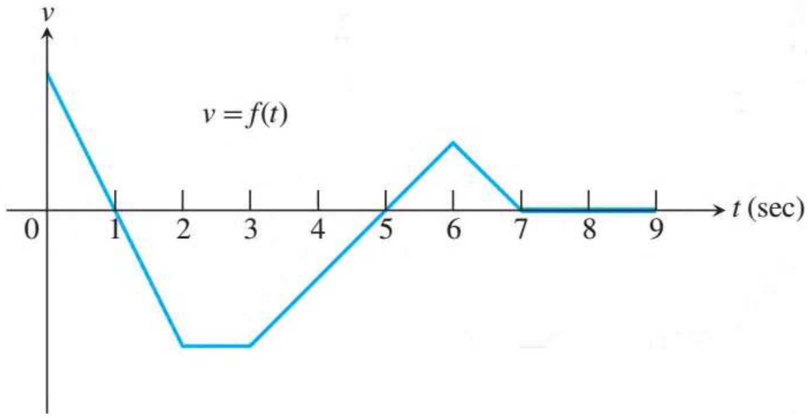
Do Now:

The accompanying figure shows the velocity $v = f(t)$ of a particle moving on a coordinate line



a. When does the particle move forward? Move backward?

forward: $(0,1) \cup (5,7)$ since $v(t) > 0$

backwards: $(1,5)$ since $v(t) < 0$

b. When is the particle's acceleration positive? Negative? Zero?

$a(t) = v'(t)$

$a(t) = v'(t) > 0$ on $(3,6)$

$a(t) = v'(t) < 0$ on $(0,1) \cup (6,7)$

$a(t) = v'(t) = 0$ on $(2,3) \cup (7,9)$

c. On what interval does the particle move at its greatest speed?

$|v(t)|$ $(2,3)$

d. When does the particle stand still for more than an instant?

$v(t) = 0$ $(7,9)$

e. On what interval(s) is the particle's speed increasing? Speed decreasing? Justify your answer.

accelerating: $v(t)$ & $a(t)$ same sign $(1,2) \cup (5,6)$

decelerating: $v(t)$ & $a(t)$ differ in sign $(0,1) \cup (3,5) \cup (6,7)$

Class Work:

1. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 12t + 5$.

   a. Find the velocity and acceleration of the particle at any time t.

   $v(t) = 3t^2 - 12$

   $a(t) = 6t$

   b. Find all values of t for which the particle is at rest.

   $v(t) = 3t^2 - 12 = 0$

   $t^2 = 4$

   $t = \pm 2$

   2 seconds

   c. Find the speed of the particle when its acceleration is zero.

   $a(t) = 6t = 0$

   $t = 0$

   $v(0) = -12$

   speed at $t=0$ is 12

   d. Is the particle moving toward the origin or away from the origin when $t = 3$? Justify your answer.

   The particle is moving toward the origin

   $s(0) = 5$

   $s(2) = -11$

   $s(3) = -4$

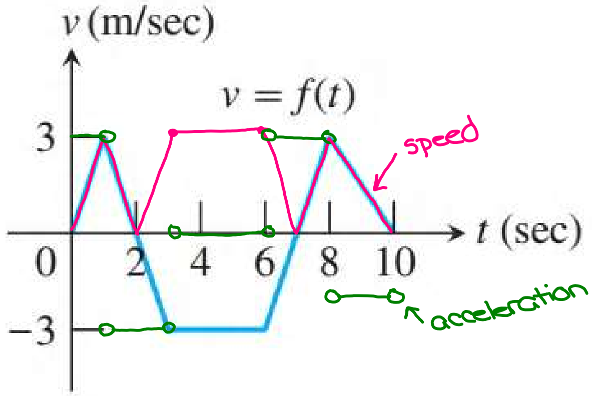
   $v(3) = 15 \rightarrow$ moving right from $s(3) = -4$

   e. Is the particle's speed increasing or decreasing at $t = 3$? Justify your answer.

   $v(3) = 15$, $a(3) = 18$ } Same sign/direction

   speed is increasing

2. The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)\,(m/\sec)$ of a body moving along a coordinate line.



a. When does the body reverse direction? Justify your answer.

v(t) changes sign → t = 2, 7

b. When (approximately) is the body moving at a constant speed? a(t) = v′(t) = 0

(3, 6)

c. Graph the body's speed for $0 \le t \le 10$ on the graph above in colored pencil.

d. Graph the acceleration (where defined) on the graph above in another color.

e. Is the body's speed increasing or decreasing at $t = 1.5$? At $t = 5$? At $t = 7.5$? Justify your answers.

t = 1.5	t = 5	t = 7.5
v(1.5) > 0	v(5) < 0	v(7.5) > 0
a(1.5) < 0	a(5) = 0	a(7.5) > 0
decreasing	neither → constant	increasing

3. On Earth, if you shoot a paper clip 64 ft straight up into the air with a rubber band, the paper clip will be $s(t) = 64t - 16t^2$ feet above your hand at t sec after firing.

a. Find ds/dt and d^2s/dt^2.

$\frac{ds}{dt} = 64 - 32t$

$\frac{d^2s}{dt^2} = -32$

b. How long does it take the paper clip to reach its maximum height? What is the maximum height?

$\frac{ds}{dt} = 64 - 32t = 0$

$t = 2$

$S(2) = 128 - 64 = 64$

2 seconds to reach 64 ft

c. With what velocity does it leave your hand?

$\left.\frac{ds}{dt}\right|_{t=0} = 64$ ft/sec

d. On the moon, the same force will send the paper clip to a height of $m(t) = 64t - 2.6t^2$ ft in t sec. About how long will it take the paper clip to reach its maximum height, and how high will it go?

$m'(t) = 64 - 5.2t = 0$

$64 = 5.2t$

$t \approx 12.307$ seconds

$m(12.307) = 393.846$ ft

e. Compare the acceleration on earth to the acceleration on the moon. What does this, in conjunction with your answers to b and d above, tell you about the difference of gravitational forces on Earth and the moon?

acceleration on earth: -32 ft/sec

acceleration on moon: -5.2 ft/sec

stronger gravitational force on earth than on the moon!