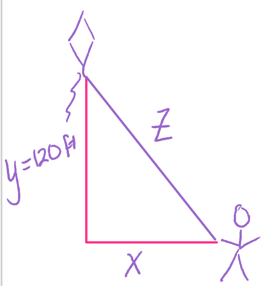
Name *Answer Key* Date________________

Calc I H – 2.6 day 5 – Related Rate Practice **Period_______**

Solve the following problems fully, **showing each step (including a sketch)** of your work **NEATLY.** Be sure to include units of measurement in your final answer! *Write your answer in terms of* π **and** *round your final answers to the nearest thousandth.*

1.) A child is flying a kite. If the kite is 120 feet above the child's hand and the wind is blowing on a horizontal course at 4 feet per second, how fast is the child paying out cord when 200 feet of the cord is out? (Assume the cord forms a straight line)



$\frac{dx}{dt} = 4\frac{ft}{sec}$, $y = 120$ ft find $\frac{dz}{dt}$ when $z = 200$ ft.

Constant: $y = 120$ ft

Varies: x & z

$\frac{d}{dt}(x^2 + 120^2 = z^2)$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$

when $z = 200 \Rightarrow x^2 + 120^2 = 200^2$

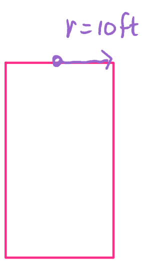
$\sqrt{x^2} = \sqrt{25{,}600}$

$x = 160$ $(x \neq -160)$

$\frac{dz}{dt} = \frac{160}{200}(4)$

$\boxed{\frac{dz}{dt} = \frac{16}{5}\frac{ft}{sec} = 3.2\frac{ft}{sec}}$

2.) Water flows at a rate of 5 cubic feet per minute into a cylinder with a radius of 10 feet. How fast is the water level rising?



$\frac{dV}{dt} = 5\frac{ft^3}{min}$, $r = 10$ ft Find $\frac{dh}{dt}$.

Constant: r

varies: V, h

$V = \pi r^2 h = \pi(10)^2 h$

$\frac{d}{dt}(V = 100\pi h)$

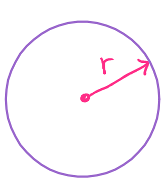
$\frac{dV}{dt} = 100\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV/dt}{100\pi}$

$\frac{dh}{dt} = \frac{5}{100\pi}$

$\boxed{\frac{dh}{dt} = \frac{1}{20\pi}\frac{ft}{min} \approx .016\frac{ft}{min}}$

3.) Air is filling a spherical balloon at the rate of 7 cubic inches per minute. When the radius is 10 inches, how fast is the radius increasing?



$\frac{dV}{dt} = 7\frac{in^3}{min}$, Find $\frac{dr}{dt}$ when $r = 10$ in.

Constant: none

varies: V, r

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

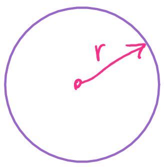
$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

$\frac{dr}{dt} = \frac{7}{4\pi(10)^2}$

$\boxed{\frac{dr}{dt} = \frac{7}{400\pi}\frac{in}{min} \approx .006\frac{in}{min}}$

4.) Oil is spilled into a lake forming a circular film on the surface of the water. If the radius of the circle increases at a rate of 8 meters per minute, how fast is the area of the circle increasing when the radius is 90 meters?



$\frac{dr}{dt} = 8\ \frac{m}{min}$ Find $\frac{dA}{dt}$ when $r = 90m$

Constant: none
Varies: A, r

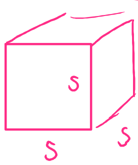
$\frac{d}{dt}(A = \pi r^2)$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(90)(8)$

$\boxed{\frac{dA}{dt} = 1440\pi\ \frac{m^2}{min} \approx 4523.893\ \frac{m^2}{min}}$

5.) A cube of ice is melting. The sides of the cube are decreasing at the constant rate of 3 inches per hour. How fast is the volume of the cube decreasing when each side is 6 inches?



$\frac{ds}{dt} = -\frac{3in}{hr}$, Find $\frac{dV}{dt}$ when $S = 6in.$

Constant: none
Varies: V, S

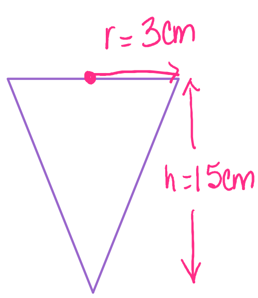
$\frac{d}{dt}(V = s^3)$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3(6)^2(-3)$

$\boxed{\frac{dV}{dt} = -324\ \frac{in^3}{hr}}$

6.) Water is being drained from a conical funnel (with vertex down) at a rate of 5 cubic centimeters per second. If the height of the cup is 15 cm and the diameter of the cup at the top is 6 cm, how fast is the level of liquid falling when the depth of the liquid is 3 cm?



$\frac{dV}{dt} = -5\ \frac{cm^3}{sec}$, Find $\frac{dh}{dt}$ when $h = 3cm.$

Constant: none
Varies: r, h, V

$\frac{r}{h} = \frac{3}{15}$

$r = \frac{1}{5}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{5}h\right)^2 h$

$\frac{d}{dt}\left(V = \frac{\pi}{75}h^3\right)$

$\frac{dV}{dt} = \frac{\pi}{25}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{25\, dV/dt}{\pi h^2}$

$\frac{dh}{dt} = \frac{25(-5)}{\pi(3)^2}$

$\boxed{\frac{dh}{dt} = \frac{-125}{9\pi}\ \frac{cm}{sec} \approx 4.421\ \frac{cm}{sec}}$