Name Answer Key **Date**_____

Calc I H – 2.6 day 6 – Related Rate Practice **Period**_____

Solve the following problems fully, **showing each step (including a sketch)** of your work **NEATLY.** Be sure to include units of measurement in your final answer! *Write your answer in terms of* π **and** *round your final answers to the nearest thousandth.*

1.) Oil from a leaking oil tanker radiates outward in the form of a circular film on the surface of the water. If the radius of the circle increases at the rate of 3 meters per minute, how fast is the area of the circle increasing when the radius is 200 meters?

$\frac{dr}{dt} = 3\frac{m}{min}$ Find $\frac{dA}{dt}$ when $r = 200m$.

Constant: none
Varies: A, r

$\frac{d}{dt}(A = \pi r^2)$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(200)(3)$

$\boxed{\frac{dA}{dt} = 1200\pi \frac{m^2}{min} \approx 3769.911 \frac{m^2}{min}}$

2.) Oil spilled from a tanker spreads in a circle whose circumference increases at a rate of 40ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 100π ft?

$\frac{dC}{dt} = 40\frac{ft}{sec}$ Find $\frac{dA}{dt}$ when $C = 100\pi$ ft. Need: r

Constant: none
Varies: A, C, r

$\frac{d}{dt}(A = \pi r^2)$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Need: $\frac{dr}{dt}$ $\Rightarrow \frac{d}{dt}(C = 2\pi r)$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dC/dt}{2\pi} = \frac{40}{2\pi} = \frac{20}{\pi}\frac{ft}{sec}$

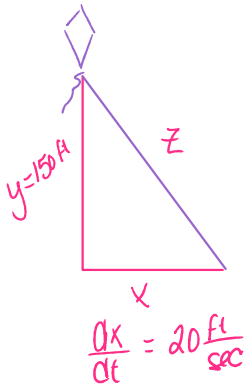
$C = 2\pi r$
$100\pi = 2\pi r$
$50ft = r$

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(50)\left(\frac{20}{\pi}\right)$

$\boxed{\frac{dA}{dt} = 2000\frac{ft^2}{sec}}$

3.) A boy is flying a kite at a height of 150 ft. If the kite moves horizontally away from the boy at 20ft/sec, how fast is the string being pulled out when the kite is 250 ft from him?



$\frac{dx}{dt} = 20\frac{ft}{sec}$

$y = 150\,ft$, $\frac{dx}{dt} = 20\frac{ft}{sec}$ Find $\frac{dz}{dt}$ when $z = 250$ ft.

Constant: $y = 150$ ft
Varies: x, z

$\frac{d}{dt}(x^2 + 150^2 = z^2)$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$

Need: x

$x^2 + 150^2 = 250^2$
$x^2 = 40{,}000$
$x = 200$ $(x \neq -200)$

$\frac{dz}{dt} = \frac{200}{250}(20)$

$\boxed{\frac{dz}{dt} = 16\frac{ft}{sec}}$

4.) A liquid is flowing into a vertical cylindrical tank of radius 6 ft at the rate of 8 ft^3/min. How fast is the surface rising?

$r = 6\,ft \quad \frac{dV}{dt} = 8\,\frac{ft^3}{min}$ Find $\frac{dh}{dt}$.

Const: $r = 6$ ft
Varies: V, h

$V = \pi r^2 h$

$\frac{d}{dt}(V = 36\pi h)$

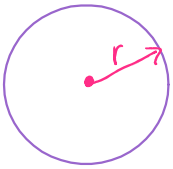
$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV/dt}{36\pi}$

$\frac{dh}{dt} = \frac{8}{36\pi}$

$\boxed{\frac{dh}{dt} = \frac{2}{9\pi}\,\frac{ft}{min} \approx .071\,\frac{ft}{min}}$

5.) A spherical balloon is inflating at a rate of 27π in^3/sec. How fast is the radius of the balloon increasing when the radius is 3 inches?



$\frac{dV}{dt} = 27\pi\,\frac{in^3}{sec}$, Find $\frac{dr}{dt}$ when $r = 3$ in.

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

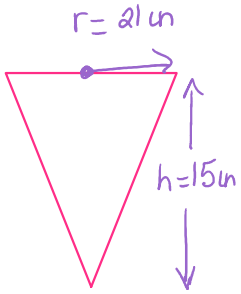
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

$\frac{dr}{dt} = \frac{27\pi}{4\pi (3)^2}$

$\boxed{\frac{dr}{dt} = \frac{3}{4}\,\frac{in}{sec}}$

6.) An inverted conical container has a diameter of 42 inches and a depth of 15 inches. If water is flowing out of the vertex of the container at a rate of 35π in^3/sec, how fast is the depth of the water dropping when the height is 5 inches?



$\frac{dV}{dt} = -35\pi\,\frac{in^3}{sec}$ Find $\frac{dh}{dt}$ when $h = 5$ in.

Const: none
Varies: r, h, V

$\frac{r}{h} = \frac{21}{15}$

$r = \frac{7}{5}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{7}{5}h\right)^2 h$

$\frac{d}{dt}\left(V = \frac{49\pi}{75}h^3\right)$

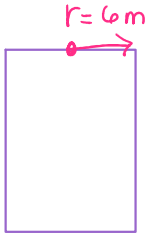
$\frac{dV}{dt} = \frac{49\pi}{25}h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{25\,dV/dt}{49\pi h^2}$

$\frac{dh}{dt} = \frac{25(-35\pi)}{49\pi (5)^2}$

$\boxed{\frac{dh}{dt} = -\frac{5}{7}\,\frac{in}{sec} \approx -.714\,\frac{in}{sec}}$

7.) A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of $108\pi\ m^3/sec$. How fast is the height increasing?



$r = 6m,\ \frac{dV}{dt} = 108\pi \frac{m^3}{sec}$ Find $\frac{dh}{dt}$

Const: $r = 6m$

varies: h, V

$V = \pi r^2 h$

$\frac{d}{dt}(V = 36\pi h)$

$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV/dt}{36\pi}$

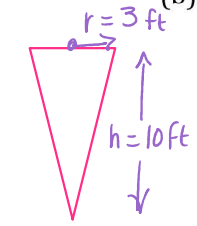
$\frac{dh}{dt} = \frac{108\pi}{36\pi}$

$\boxed{\frac{dh}{dt} = 3 \frac{m}{sec}}$

8.) Water is being withdrawn from a conical reservoir 3 feet in radius and 10 feet deep at $4\ ft^3/min$.

(a) How fast is the surface falling when the depth of the water is 6 feet?

(b) How fast is the radius of this surface diminishing?



$\frac{r}{h} = \frac{3}{10}$

$r = \frac{3}{10}h$

a) $\frac{dV}{dt} = -4\ ft^3/min$ Find, $\frac{dh}{dt}$ when $h = 6\ ft$

Const: none

varies: r, h, V

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{3}{10}h\right)^2 h$

$\frac{d}{dt}\left(V = \frac{3\pi}{100}h^3\right)$

$\frac{dV}{dt} = \frac{9\pi}{100}h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{100\, dV/dt}{9\pi h^2}$

$\frac{dh}{dt} = \frac{100(-4)}{9\pi(6)^2}$

$\boxed{\frac{dh}{dt} = \frac{-100}{81\pi}\frac{ft}{min} \approx -.393\frac{ft}{min}}$

b) Find $\frac{dr}{dt}$ when $h = 6\ ft$.

$\frac{d}{dt}\left(r = \frac{3}{10}h\right)$

$\frac{dr}{dt} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{3}{10}\left(\frac{-100}{81\pi}\right)$

$\boxed{\frac{dr}{dt} = \frac{-10}{27\pi}\frac{ft}{min} \approx -.118\frac{ft}{min}}$

Answers

1) $1200\pi m^2/\min \approx 3769.911 m^2/\min$	2) $2000\, ft^2/\sec$	3) $16\, ft/\sec$
4) $\frac{2}{9\pi}\, ft/\min \approx 0.071\, ft/\min$	5) $\frac{3}{4}\, in/\sec$	6) $-\frac{5}{7}\, in/\sec \approx -0.714\, in/\sec$
7) 3m/sec	8a) $-\frac{100}{81\pi}\, ft/\min \approx -0.393\, ft/\min$	8b) $-\frac{10}{27\pi}\, ft/\min \approx -0.118\, ft/\min$