Name_____ **Date**_____

Calc I H – 2.6 Intro HW – Differentiating with Respect to Different Variables **Period**_____

Notes:

- When you differentiate and there are multiple variables, you must show what variables you are differentiating and what variables you are differentiating with respect to.

- When we differentiate *y* with respect to *x* we write $\frac{dy}{dx}$.
  The numerator identifies the variable we differentiate.
  The denominator identifies the variable we differentiate with respect to.

- You can also differentiate with respect to variables that are not in the equation.

Examples: Differentiate with respect to *t*.
(This means your denominator will be dt, and your numerator identifies the variable that you differentiated.)

1. $2y = 3x$

$$\frac{d}{dt}(2y = 3x)$$

$$2\frac{dy}{dt} = 3\frac{dx}{dt}$$

2. $4x^2 + 3y^2 = 6$

$$\frac{d}{dt}\left(4x^2 + 3y^2 = 6\right)$$

$$8x\frac{dx}{dt} + 6y\frac{dy}{dt} = 0$$

3. Find $\frac{dy}{dt}$. *Differentiate first and then solve for the given symbol.*

$$4xy = 16$$

$$\frac{d}{dt}(4xy = 16)$$

$$4y\frac{dx}{dt} + 4x\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{y\frac{dx}{dt}}{x}$$

4. Evaluate $\frac{dx}{dt}$ for $3x^2 + y^3 = 19$ when $x = 3,\ y = -2,\ \frac{dy}{dt} = 6$.

$$\frac{d}{dt}\left(3x^2 + y^3 = 19\right)$$

$$6x\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y^2\frac{dy}{dt}}{2x}$$

$$\frac{dx}{dt}(3,-2) = -\frac{(-2)^2(6)}{2(3)} = -4$$

Your turn! Try each of the examples below.

1. Given $y = 2x^2 - 5xy + 6y^2$, find

a) $\frac{dy}{dx}$ $\frac{d}{dx}(y = 2x^2 - 5xy + 6y^2)$

$\frac{dy}{dx} = 4x - 5y - 5x\frac{dy}{dx} + 12y\frac{dy}{dx}$

$\frac{dy}{dx}(1 + 5x - 12y) = 4x - 5y$

$\boxed{\frac{dy}{dx} = \frac{4x-5y}{1+5x-12y}}$

b) $\frac{dx}{dy}$ $\frac{d}{dy}(y = 2x^2 - 5xy + 6y^2)$

$1 = 4x\frac{dx}{dy} - 5y\frac{dx}{dy} - 5x + 12y$

$1 + 5x - 12y = (4x - 5y)\frac{dx}{dy}$

$\boxed{\frac{dx}{dy} = \frac{1+5x-12y}{4x-5y}}$

2. Given $A = L \cdot W$, find

a) $\frac{dA}{dL}$ $\frac{d}{dL}(A = LW)$

$\boxed{\frac{dA}{dL} = W + L\frac{dW}{dL}}$

b) $\frac{dA}{dW}$ $\frac{d}{dW}(A = LW)$

$\boxed{\frac{dA}{dW} = W\frac{dL}{dW} + L}$

3. Given $V = \frac{4}{3}\pi r^3$, find $\frac{dV}{dr}$.

$\frac{d}{dr}(V = \frac{4}{3}\pi r^3)$ $\frac{dV}{dr} = 3(\frac{4}{3})\pi r^2$ $\boxed{\frac{dV}{dr} = 4\pi r^2}$

4. Given $m^2 = n^2 + p^2$, evaluate $\frac{dp}{dt}$ when $m = 5$, $n = -3$, $p = -4$, $\frac{dm}{dt} = -1$, $\frac{dn}{dt} = -3$.

$\frac{d}{dt}(m^2 = n^2 + p^2)$

$2m\frac{dm}{dt} = 2n\frac{dn}{dt} + 2p\frac{dp}{dt}$

$\frac{dp}{dt} = \frac{m\frac{dm}{dt} - n\frac{dn}{dt}}{p}$

$\frac{dp}{dt}\Big|_{(5,-3,-4)} = \frac{5(-1) - (-3)(-3)}{-4}$

$= \frac{-5-9}{-4} = \boxed{\frac{13}{4}}$

5. Given $V = \frac{1}{3}\pi r^2 h$, evaluate $\frac{dV}{dt}$ when $h = 12$, $r = 3$, $\frac{dr}{dt} = 2$, $\frac{dh}{dt} = 4$

$\frac{d}{dt}(V = \frac{1}{3}\pi r^2 h)$

$\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$

$\frac{dV}{dt}\Big|_{(12,3)} = \frac{2}{3}\pi(3)(12)(2) + \frac{1}{3}\pi(3)^2(4)$

$= 48\pi + 12\pi = \boxed{60\pi}$

6. Find $\frac{dz}{dt}$, where z is the hypotenuse of a right triangle with legs equal to x and y, when $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 2$, $z = 50$, $x = 30$ $\frac{d}{dt}(x^2 + y^2 = z^2)$ $\Rightarrow 30^2 + y^2 = 50^2$ $y^2 = 1600$, $y = 40$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

$\frac{dz}{dt}\Big|_{(30,40,50)} = \frac{30(0) + 40(2)}{50}$

$= \frac{80}{50} = \boxed{\frac{8}{5}}$