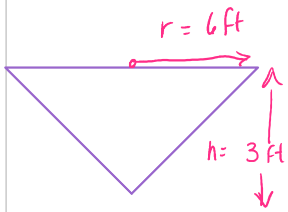
Name Answer Key Date

Calc I H – 2.6– Review **Period**

Solve the following problems fully, **showing each step (including a sketch)** of your work **NEATLY.** Be sure to include units of measurement in your final answer! *Write your answer in terms of π* **and** *round your final answers to the nearest thousandth.*

1.) *Cone problem*
A conical tank (with vertex down) is 3 feet deep. If water is flowing into the tank at the rate of 3 cubic feet/minute, find the rate of change of depth of water when the water is 1 foot deep. The radius of the conical tank on the top is given by 6 feet.



$\frac{r}{h} = \frac{6}{3}$

$r = 2h$

$\frac{dV}{dt} = 3\,\frac{ft^3}{min}$ Find $\frac{dh}{dt}$ when $h = 1$ ft

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}(2h)^2 h$

$\frac{d}{dt}\left(V = \frac{4\pi}{3} h^3\right)$

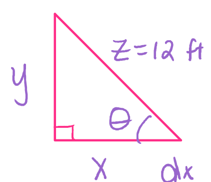
$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV/dt}{4\pi h^2}$

$\frac{dh}{dt} = \frac{3}{4\pi(1)^2}$

$\boxed{\frac{dh}{dt} = \frac{3}{4\pi}\,\frac{ft}{min} \approx .239\,\frac{ft}{min}}$

2.) *Ladder Problem*
(a) A 12 foot ladder stands upright against a vertical wall. If the lower end of the ladder is pulled away from the wall at the rate of 2 feet/second, how fast is the top of the ladder coming down the wall at the instant it is 6 feet above the ground?
(b) How fast is the angle between the ladder and the ground changing?



$\frac{dx}{dt} = 2\,\frac{ft}{sec} \rightarrow$

a) Find $\frac{dy}{dt}$ when $y = 6$ ft

$\frac{d}{dt}\left(x^2 + y^2 = 12^2\right)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$

when $y = 6$

$x^2 + 6^2 = 12^2$

$x^2 = 108$

$x = 6\sqrt{3}$

$\Rightarrow \frac{dy}{dt} = -\frac{6\sqrt{3}}{6}(2)$ $\boxed{= -2\sqrt{3}\,\frac{ft}{sec} \approx 3.464\,\frac{ft}{sec}}$

b) Find $\frac{d\theta}{dt}$

$\frac{d}{dt}\left(\cos\theta = \frac{x}{12}\right)$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{12}\frac{dx}{dt}$

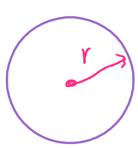
hyp / opp

$\frac{d\theta}{dt} = \frac{-1}{12}\frac{dx}{dt}\csc\theta$

$= \frac{-1}{12}(2)\left(\frac{12}{6}\right)$

$= \boxed{-\frac{1}{3}\,\frac{r}{sec}}$

3.) *Balloon Problem*
Air is being pumped into a spherical balloon at the rate of 3.2 cubic feet per minute. Find the rate of change of the radius when the radius is 1.5 feet.



$\frac{dV}{dt} = 3.2\,\frac{ft^3}{min}$ Find $\frac{dr}{dt}$ when $r = 1.5$ ft.

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

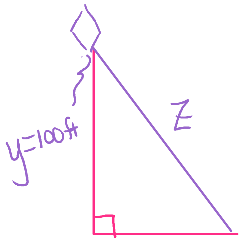
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

$\frac{dr}{dt} = \frac{3.2}{4\pi(1.5)^2}$

$\boxed{\frac{dr}{dt} = \frac{16}{45\pi}\,\frac{ft}{min} \approx .113\,\frac{ft}{min}}$

4.) *Kite Problem*
A kite 100 feet high is being blown horizontally at the rate of 8 feet/second when 300 feet of string is out. How fast is string running out? (Ignore air effect)



$\frac{dx}{dt} = 8\,\frac{ft}{sec}$

Find $\frac{dz}{dt}$ when $z = 300$ ft

$\frac{d}{dt}\left(x^2 + 100^2 = z^2\right)$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt} \Rightarrow \frac{dz}{dt} = \frac{200\sqrt{2}}{300}(8)$

Need: x

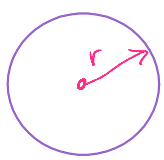
$x^2 + 100^2 = 300^2$

$x^2 = 80{,}000$

$x = 200\sqrt{2}$ ft

$\boxed{\frac{dz}{dt} = \frac{16\sqrt{2}}{3}\,\frac{ft}{sec} \approx 7.542\,\frac{ft}{sec}}$

5.) *Balloon Problem*
Air is leaking from a spherical balloon at the rate of 1.4 cubic inches/minute. Find the rate of change of the radius when the radius is 5 inches.



$\frac{dV}{dt} = -1.4\,\frac{in^3}{min}$ Find $\frac{dr}{dt}$ when $r = 5$ in

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

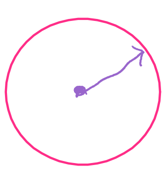
$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

$\frac{dr}{dt} = \frac{-1.4}{4\pi(5)^2}$

$\boxed{\frac{dr}{dt} = \frac{-.7}{500\pi}\,\frac{in}{min} \approx .004\,\frac{in}{min}}$

6.) *Ripple Problem*
A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at the constant rate of 1 foot/second. When the radius is 4 feet, at what rate is the total area of the disturbed water changing?

$\frac{dr}{dt} = 1\,\frac{ft}{sec}$ Find $\frac{dA}{dt}$ when $r = 4$ ft



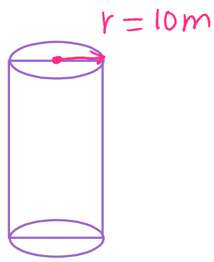
$\frac{d}{dt}\left(A = \pi r^2\right)$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(4)(1)$

$\boxed{\frac{dA}{dt} = 8\pi\,\frac{ft^2}{sec} \approx 25.133\,\frac{ft^2}{sec}}$

7.) *Cylinder Problem*
A cylindrical tank with a radius of 10 meters is filling with fluid at the rate of 150π m^3/second. How fast is the height of the liquid changing?



$\frac{dV}{dt} = 150\pi \frac{m^3}{sec}$ Find $\frac{dh}{dt}$

$V = \pi r^2 h$ Note: In a cylinder, the radius is constant!

$V = \pi (10)^2 h$

$\frac{d}{dt}(V = 100\pi h)$

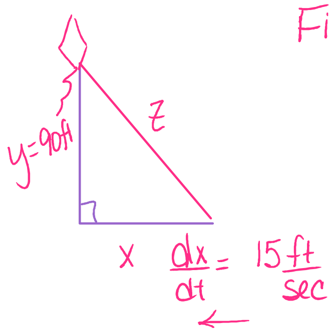
$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV/dt}{100\pi}$

$= \frac{150\pi}{100\pi}$

$\boxed{\frac{dh}{dt} = \frac{3}{2}\frac{m}{s} = 1.5\frac{m}{s}}$

8.) *Kite Problem*
A boy is flying a kite at a height of 90 feet. If the kite moves horizontally only at the rate of 15 feet/second, how fast is the string running out when the kite is 150 feet away from him?



Find $\frac{dz}{dt}$ when $z = 150$ ft

$(x^2 + 90^2 = z^2)$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$

Need: x

$x^2 + 90^2 = 150^2$

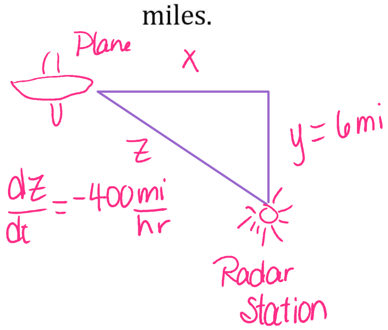
$x^2 = 14,400$

$x = 120$

$\frac{dz}{dt} = \frac{\cancel{120}}{\cancel{150}}(\cancel{15})$

$\boxed{\frac{dz}{dt} = 12\frac{ft}{sec}}$

9.) *Plane Problem*
An airplane is flying at an altitude of 6 miles on a flight path that will take it directly over a radar tracking station. If the distance between the radar and the plane is decreasing at the rate of 400 miles/hour, find the speed of the plane when the distance between the radar and the plane is 10 miles.



Find $\frac{dx}{dt}$ when $z = 10$ mi

$\frac{d}{dt}(x^2 + 6^2 = z^2)$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}$

Need x

$x^2 + 6^2 = 10^2$

$x^2 = 64$

$x = 8$ mi

$\frac{dx}{dt} = \frac{10}{8}(-400)$

$\frac{dx}{dt} = -500\frac{mi}{hr}$

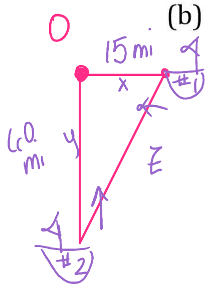
$\boxed{\text{Speed of plane is } 500\frac{mi}{hr}}$

10.) *Ship Problem*

Ship #1 is 15 miles east of the point O and moving west at the rate of 20 miles/hour. Ship #2 is 60 miles south of the point O and moving north at the rate of 15 miles/hour.

(a) Are they approaching or separating after 1 hour?

(b) Are they approaching or separating after 3 hours?



$\frac{dx}{dt} = 20 \frac{mi}{hr}$

$\frac{dy}{dt} = 15 \frac{mi}{hr}$

a) Find $\frac{dz}{dt}$ after 1 hour

In 1hr, $x = 5mi$ and $\frac{dx}{dt} = 20 \frac{mi}{hr}$

$y = 60-15 = 45mi$ and $\frac{dy}{dt} = -15\frac{mi}{hr}$

Then $x^2 + y^2 = z^2$

$5^2 + 45^2 = z^2$

$z = 5\sqrt{82}$

$45^2 + 15^2 = z^2$

$z = 15\sqrt{10}$

$\frac{d}{dt}\left(x^2 + y^2 = z^2\right)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{5(20) + 45(-15)}{5\sqrt{82}} = \frac{-115}{\sqrt{82}} \frac{m}{sec}$

* Since $\frac{dz}{dt} < 0$, the distance between them is decreasing - approaching

b) Find $\frac{dz}{dt}$ after 3hrs

In 3hrs, $x = 45mi$, $\frac{dx}{dt} = 20\frac{mi}{hr}$

$y = 15mi$, $\frac{dy}{dt} = -15\frac{mi}{hr}$

$\frac{dz}{dt} = \frac{45(20) + 15(-15)}{15\sqrt{10}}$

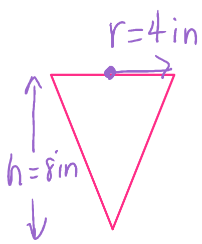
$= \frac{45}{\sqrt{10}}$ * Since $\frac{dz}{dt} > 0$ - distance increasing - Separating

11.) *Cone Problem*

Water is running out of a conical funnel at the rate of 1 inch³/second. If the radius of the base of the funnel is 4 inches and the height is 8 inches:

(a) Find the rate at which the water level is dropping when it is 2 inches from the top.

(b) Find the rate at which the radius is decreasing.



$\frac{r}{h} = \frac{4}{8}$

$r = \frac{1}{2}h$

$\frac{dV}{dt} = -1 \frac{in^3}{sec}$

a) Find $\frac{dh}{dt}$ when $h = 6in$ (2in from top)

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$\frac{d}{dt}\left(V = \frac{\pi}{12} h^3\right)$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4\,dV/dt}{\pi h^2}$

$\frac{dh}{dt} = \frac{4(-1)}{\pi(6)^2}$

$$\boxed{\frac{dh}{dt} = \frac{-1}{9\pi} \frac{in}{sec} \approx .035 \frac{in}{sec}}$$

b) Find $\frac{dr}{dt}$

$\frac{d}{dt}\left(r = \frac{1}{2}h\right)$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{1}{2}\left(\frac{-1}{9\pi}\right)$

$$\boxed{\frac{dr}{dt} = \frac{-1}{18\pi} \frac{in}{sec} \approx .018 \frac{in}{sec}}$$