Name Answer Key Date________

Calc I H – 3.1 – 3.2 PA Review Period ________

1. Find any critical numbers of the function.

a. $f(x)=\frac{4x}{1-x}$ $4-4x+4x$

$f'(x)=\frac{4(1-x)-(-1)(4x)}{(1-x)^2}=\frac{4}{(1-x)^2}$

$f'(x)\neq 0$

$f'(x)$ und $\Rightarrow (1-x)^2=0,\ x=1$

$x=1$ NOT in domain of f. No crit #'s

b. $f(x)=x^3+x^2-8x+5$

$f'(x)=3x^2+2x-8=0$, ~~$f'(x)$ und~~ not possible

$(3x-4)(x+2)=0$

$\boxed{\text{Crit \#'s: } x=4/3,\ -2}$

2. Determine the extrema for $f(x)=x^3-12x$ on the interval $[0,4]$.

$f'(x)=3x^2-12=0$ ~~$f'(x)$ und~~ not possible

$3(x^2-4)=0$

$3(x+2)(x-2)=0$

$x=\pm 2$ $x=-2$ NOT in interval

$f(0)=0$ NOT extrema

$f(2)=-16$ Abs Min: $(2,-16)$

$f(4)=16$ Abs Max: $(4,16)$

3. Determine the extrema for $g(x)=\frac{2x}{x^2+1}$ on the interval $[-2,2]$.

$f'(x)=\frac{2(x^2+1)-2x(2x)}{(x^2+1)^2}=\frac{-2x^2+2}{(x^2+1)^2}$

$f'(x)=0=-2x^2+2$ ~~$f'(x)$ und~~ not possible

$0=x^2-1$

$0=(x+1)(x-1)\Rightarrow x=\pm 1$

$f(-2)=-4/5$ NOT extrema

$f(-1)=-1$ Abs Min: $(-1,-1)$

$f(1)=1$ Abs Max: $(1,1)$

$f(2)=\frac{4}{5}$ NOT extrema

4. If possible, apply Rolle's Theorem to find all values of c in the interval such that $f'(c)=0$.

$f(x)=x^2-3x+2,\ [1,2]$

$f(x)$ everywhere cont & diff, $\therefore f(x)$ cont on $[1,2]$ and diff on $(1,2)$.

$\left.\begin{array}{l} f(1)=0 \\ f(2)=0 \end{array}\right\}$ Since $f(1)=f(2)$ Rolle's Thm applies.

$f'(x)=2x-3$

$f'(c)=2c-3=0$

$\boxed{c=3/2}$

5. If possible, apply Rolle's Theorem to find all values of c in the interval such that $f'(c)=0$.

$f(x)=\frac{x^2-1}{x},\ [-1,1]$

$f(x)$ cont & diff on $(-\infty,0)\cup(0,\infty)$

Since $f(x)$ not diff at $x=0$, Rolle's Thm does not apply on $[-1,1]$ which includes $x=0$.

6. If possible apply the Mean Value Theorem to find all values of c in the interval such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

a. $f(x) = x(x^2 - x - 2), [-1, 1]$

$f(x) = x^3 - x^2 - 2x$

$f(x)$ is everywhere cont & diff, ∴ cont on $[-1, 1]$ and diff on $(-1, 1)$. MVT applies.

$f(-1) = 0$

$f(1) = -2$

$f'(x) = 3x^2 - 2x - 2$

$\dfrac{f(1) - f(-1)}{1 - (-1)} = f'(c)$

$\dfrac{-2}{2} = 3c^2 - 2c - 2$

$0 = 3c^2 - 2c - 1$

$0 = (3c + 1)(c - 1)$

$\boxed{c = -1/3}$, ~~$c = 1$~~ endpt

b. $f(x) = \dfrac{x+1}{x}, \left[\dfrac{1}{2}, 2\right]$

$f(x)$ is cont & diff on $(-\infty, 0) \cup (0, \infty)$. ∴ $f(x)$ cont on $[\frac{1}{2}, 2]$ and diff on $(\frac{1}{2}, 2)$. MVT applies.

$f(\frac{1}{2}) = 3$

$f(2) = \frac{3}{2}$

$f'(x) = \dfrac{x - (x+1)}{x^2} = \dfrac{-1}{x^2}$

$\dfrac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \dfrac{-1}{c^2}$

$\dfrac{\frac{3}{2} - 3}{\frac{3}{2}} = -\dfrac{1}{c^2}$

$-1 = -\dfrac{1}{c^2}$

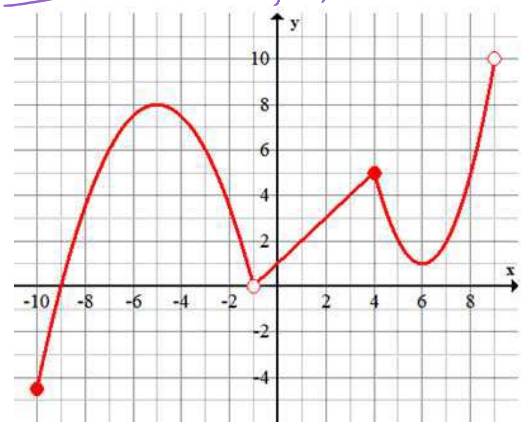
$c^2 = 1 \quad c = \pm 1$

$c = -1$ Not in interval

$\boxed{c = 1}$

7. List all the critical values for the graph below.

Crit #'s : $x = -5, 4, 6$



8. Label each point below as absolute or relative maximum or minimum or neither.

$(-10, 4.5)$ Abs min

$(-5, 8)$ Rel max

$(-1, 0)$ Neither

$(4, 5)$ Rel max

$(6, 1)$ Rel min

$(9, 10)$ Neither

Solutions:

1. a) No crit #s b) $x = \frac{4}{3}, -2$	2. $x = -2$ is not in the interval Endpt: (0,0) NOT extrema Crit #: Abs. Min. (2,-16) Endpt: Abs. Max. (4,16) 3. Endpt: $\left(-2, -\frac{4}{5}\right)$ NOT extrema Crit #: Abs. Min. (-1, -1) Crit #: Abs. Max. (1, 1) Endpt: $\left(2, \frac{4}{5}\right)$ NOT extrema	4. $c = \frac{3}{2}$ 5. $f(x)$ is not continuous on the given interval because of an asymptote at $x = 0$. Rolle's Theorem does not apply 6. a) $c = -\frac{1}{3}$ b) $c = 1$	7. $x = -5, \quad 4, 6$ 8. $(-10, 4.5)$ Abs. Min. $(-5, 8)$ Relative Max. $(-1, 0)$ Neither $(4, 5)$ Relative Max. $(6, 1)$ Relative Min. $(9, 10)$ Neither