

Name Answer Key

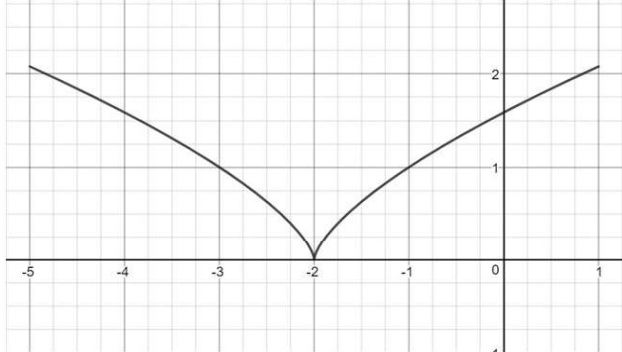
Date \_\_\_\_\_

Calc I H - 3.1 day 2 - Extrema on an Interval

Period \_\_\_\_\_

**Directions:** For each example below, find all critical numbers on the given interval and identify all absolute extrema.

1.  $f(x) = (x+2)^{\frac{2}{3}}$  on the interval  $[-5, 1]$ .



$$f'(x) = \frac{2}{3(x+2)^{1/3}}$$

Crit #'s:  
 $f'(x) \neq 0$

$f'(x)$  und when  $x = -2$

$$f(-5) = (-3)^{2/3} = \sqrt[3]{9}$$

$$f(-2) = 0$$

$$f(1) = (3)^{2/3} = \sqrt[3]{9}$$

Abs Max:

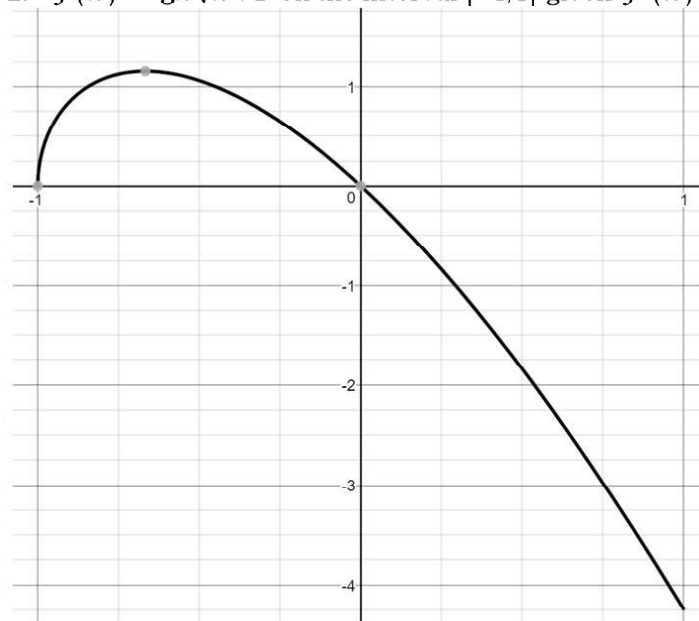
$$(-5, \sqrt[3]{9})$$

$$(1, \sqrt[3]{9})$$

Abs Min:

$$(-2, 0)$$

2.  $f(x) = -3x\sqrt{x+1}$  on the interval  $[-1, 1]$  given  $f'(x) = -\frac{9x+6}{2\sqrt{x+1}}$ .



Crit #'s:

$$f'(x) = 0$$

$$9x+6=0$$

$$x = -2/3$$

$f'(x)$  und

$$x = -1$$

$$f'(-1) = 0 \text{ NOT extrema}$$

$$f'(-2/3) = 2\sqrt{1/3} = \frac{2}{\sqrt{3}}$$

$$f'(1) = -3\sqrt{2}$$

Abs Max:

$$(-2/3, 2/\sqrt{3})$$

Abs Min:

$$(1, -3\sqrt{2})$$

3.  $h(s) = \frac{1}{s-2}$  on the interval  $[-1, 1]$ .

$$h'(s) = \frac{-1}{(s-2)^2}$$

Crit #'s:

$h'(s) \neq 0$ ,  $h'(s)$  und

when  $s-2=0$ ,  $s=2^*$  NOT in interval!

$$h(-1) = -1/3$$

$$h(1) = -1$$

Abs Max:

$$(-1, -1/3)$$

Abs Min:

$$(1, -1)$$

4.  $g(x) = -x^3 + 12x$  on the interval  $[-1, 4]$ .

Crit #'s:

$$g(x) = -3x^2 + 12 = 0, \quad g'(x) \neq \text{und}$$

$$-3(x^2 - 4) = 0$$

$$-3(x+2)(x-2) = 0$$

$$x = \pm 2 \quad \text{ONLY } x=2 \text{ on interval!}$$

$g(-1) = -11$  NOT extrema

$g(2) = 16$

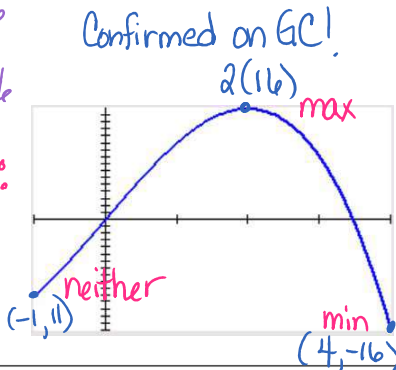
$g(4) = -16$

Abs Max:

$(2, 16)$

Abs Min:

$(4, -16)$



5.  $f(x) = x - 3\sqrt[3]{x}$  on the interval  $[-2, 3]$ .

Crit #'s:

$$f'(x) = 1 - \frac{1}{x^{2/3}} = 0$$

$$1 = \frac{1}{x^{2/3}}$$

$$x^{2/3} = 1, \quad \sqrt[3]{x^2} = 1, \quad x^2 = 1, \quad x = \pm 1$$

$f'(x)$  und when  $x^{2/3} = 0, x = 0$

$g(-2) \approx 1.780$  NOT extrema

$g(-1) = 2$

$g(0) = 0$  ← NOT extrema

$g(1) = -2$  ✓

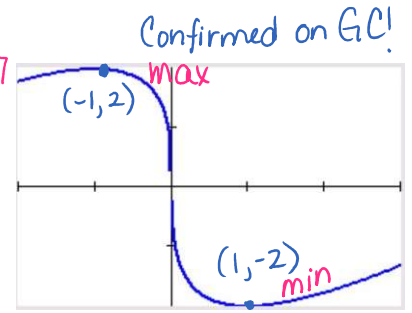
$g(3) \approx -1.327$

Abs Max:

$(-1, 2)$

Abs Min:

$(1, -2)$



6.  $h(x) = \sin^2 x + \cos x$  on the interval  $(0, 2\pi)$ .

Crit #'s:

$$h'(x) = 2\sin x \cos x - \sin x = 0 \quad h'(x) \neq \text{und}$$

$$\sin x (2\cos x - 1) = 0$$

$$\sin x = 0, \quad \cos x = \frac{1}{2}$$

$$x = \pi \quad x = \pi/3, 5\pi/3$$

$$h(\pi/3) = \sin^2(\pi/3) + \cos(\pi/3) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

$$h(\pi) = \sin^2(\pi) + \cos(\pi) = 0 - 1 = -1$$

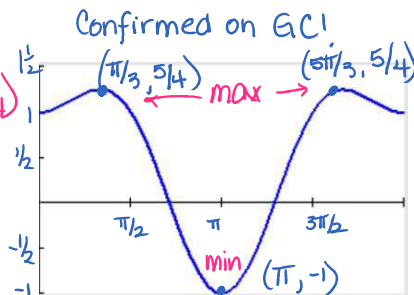
$$h(5\pi/3) = \sin^2(5\pi/3) + \cos(5\pi/3) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

Abs Max:

$(\pi/3, 5/4), (5\pi/3, 5/4)$

Abs Min:

$(\pi, -1)$



7.  $f(x) = -\frac{4}{3}x^3 - 10x^2 + 24x + 3$  on the interval

$[-7, 5]$ .

Crit #'s:

$$f'(x) = -4x^2 - 20x + 24 = 0 \quad f'(x) \neq \text{und}$$

$$-4(x^2 + 5x - 6) = 0$$

$$-4(x+6)(x-1) = 0$$

$$x = -6, 1$$

$f(-7) = -147\frac{2}{3}$  NOT extrema

$f(-6) = -213$

$f(1) = 15\frac{2}{3}$  Abs Max

$f(5) = -293\frac{2}{3}$  Abs Min

Abs Max:

$(1, 15\frac{2}{3})$

Abs Min:

$(5, -293\frac{2}{3})$

