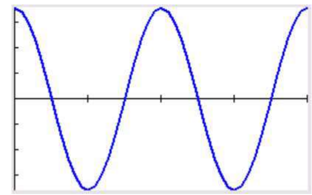
Name Answer Key　　　　　　　　　　　　　　　　Date________

Calc I H – 3.2 day 1 – Mean Value and Rolle's Theorem　　　　　　Period ________

Do Now: Find the absolute extrema of the given function on the given interval.

$f(x) = 2\cos(2x) + 1,\ [0, 2\pi]$



$f'(x) = -4\sin 2x = 0$

$2x = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$

$x = 0, \pi/2, \pi, 3\pi/2, 2\pi$

$f'(x) \neq$ und

$f(0) = 2$ abs max

$f(\pi/2) = -2$ abs min

$f(\pi) = 2$ abs max

$f(3\pi/2) = -2$ abs min

$f(2\pi) = 2$ abs max

Class Notes:

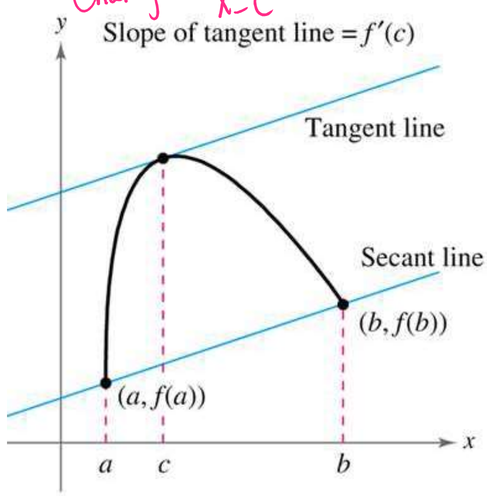
THEOREM 3.4 The Mean Value Theorem

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Instantaneous rate of change at $x = c$ → $f'(c)$

$\frac{f(b)-f(a)}{b-a}$ } Average rate of change between $x = a$ & $x = b$



Rate of Change (Velocity) Interpretation:

What does "mean" refer to in Mean Value Theorem?

Average rate of change

At $x = c$, instantaneous rate of change ($f'(c)$) equals average rate of change on interval.

Graphical Interpretation:

The slope of the secant line between $x = a$ & $x = b$ equals the slope of the tangent line at $x = c$. OR the secant line between $x = a$ & $x = b$ is parallel to the tangent line at $x = c$.

1. Given $f(x) = 5 - \frac{4}{x}$, find the value of c in the open interval $(1, 4)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ if the Mean Value Theorem applies.

$f(x)$ is continuous & diff. everywhere except at $x = 0$. $f(x)$ is continuous on $[1, 4]$ and diff on $(1, 4)$.

$f(4) = 5 - 1 = 4$

$f(1) = 5 - 4 = 1$

$f'(x) = \frac{4}{x^2}$

$f'(c) = \frac{f(4) - f(1)}{4 - 1}$

$\frac{4}{c^2} = \frac{4 - 1}{4 - 1}$

$\frac{4}{c^2} = 1$

$c^2 = 4$

$c = \pm 2$, $c = -2$ Not on interval.

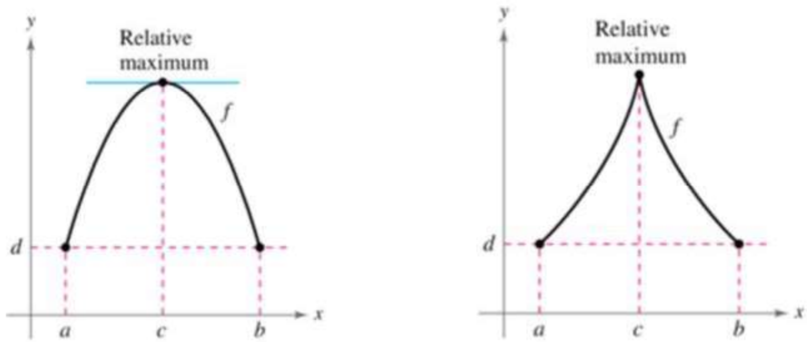
$\boxed{c = 2}$

THEOREM 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.



(a) f is continuous on $[a, b]$ and differentiable on (a, b).

(b) f is continuous on $[a, b]$.

Notes:

Special case of MVT where $f(a) = f(b)$.

2. Determine whether Rolle's Theorem can be applied to $f(x) = x^3 - 6x^2 + 11x - 2$ on the interval $[1, 3]$. If it can be applied, find all values of c on the interval such that $f'(c) = 0$. If it cannot be applied state why.

f(x) is a polynomial function & is everywhere continuous & diff.

$f(3) = 4$ Since $f(3) = f(1)$, Rolle's Thm applies.

$f(1) = 4$

$\boxed{c = \frac{6 \pm \sqrt{3}}{3}}$

$f'(x) = 3x^2 - 12x + 11 = 0$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{12}}{6} = \frac{12 \pm 2\sqrt{3}}{6} = \frac{6 \pm \sqrt{3}}{3} \approx 1.423, 2.577$

Examples using both Theorems:

3. Determine whether Rolle's Theorem can be applied to $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

f(x) is a polynomial function, ∴ everywhere cont & diff.

$f(-2) = f(2) = 8$ Rolle's Thm applies.

$f'(x) = 4x^3 - 4x = 0$

$\boxed{c = 0, \pm 1}$

$4x(x^2 - 1) = 0$

$x = 0, \pm 1$

4. Determine whether Mean Value Theorem can be applied to $f(x)=x^{\frac{1}{3}}$ on the interval $[-1,1]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

$f'(x)=\frac{1}{3x^{2/3}}$ f(x) is non-differentiable at $x=0$ on the interval $[-1,1]$.

MVT does not apply.

5. Determine whether Rolle's Theorem can be applied to $f(x)=\frac{x^2-1}{x}$ on the interval $[-2,2]$. If it can be applied, find all values of c on the interval such that $f'(c)=0$. If it cannot be applied state why.

f(x) is not continuous at $x=c$, and ∴ non-differentiable.

Rolle's Thm does not apply.

6. Determine whether Rolle's Theorem can be applied to $f(x)=(x-3)(x+1)^2$ on the interval $[-1,3]$. If it can be applied, find all values of c on the interval such that $f'(c)=0$. If it cannot be applied state why.

f(x) is a polynomial function, ∴ everywhere cont & diff. $f(-1)=0=f(3)$

Rolle's Thm applies.

$f'(x)=(x+1)^2+2(x-3)(x+1)=x^2+2x+1+2x^2-4x-6=3x^2-2x-5$

$(3x-5)(x+1)=0$

$x=5/3, \cancel{-1}$ endpt

$\boxed{c=5/3}$

7. Determine whether Mean Value Theorem can be applied to $f(x)=x^3$ on the interval $[0,1]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

f(x) is a polynomial function, ∴ everywhere cont & diff. MVT applies

$f(1)=1$, $f(0)=0$ $\quad 3c^2=\frac{1-0}{1-0}$ $\quad c=\pm\frac{1}{\sqrt{3}}$, $c=-\frac{1}{\sqrt{3}}$ not on $(0,1)$ $\quad \boxed{c=\frac{1}{\sqrt{3}}}$

$f'(x)=3x^2$ $\quad c^2=\frac{1}{3}$

8. Determine whether Rolle's Theorem can be applied to $f(x)=\sin x$ on the interval $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$. If it can be applied, find all values of c on the interval such that $f'(c)=0$. If it cannot be applied state why.

$f(x)=\sin x$ is everywhere cont & diff. $f\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=f\left(\frac{3\pi}{4}\right)$. Rolle's Thm applies.

$f'(x)=\cos x=0$ $\quad x=\frac{\pi}{2}$ $\quad \boxed{c=\frac{\pi}{2}}$

9. Find the two intercepts of $f(x)=x^2-3x+2$ and use Rolle's Theorem to show that $f'(x)=0$ at some point between the two intercepts.

f(x) is a poly. function & everywhere cont/diff.

$f(x)=0=x^2-3x+2$

$(x-2)(x-1)=0$ $\quad x=1,2$

$f(1)=f(2)=0$

By Rolle's thm, there must exist $x=c$ on $(1,2)$ where $f'(c)=0$.

10. Determine whether Mean Value Theorem can be applied to $f(x)=\sqrt{1-x^2}$ on the interval $[-1,1]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

f(x) is cont on $[-1,1]$ and diff on $(-1,1)$. ∴ MVT applies

$f(-1)=0$

$f(1)=0$

$f'(x)=\frac{-2x}{2\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}=0$ $\quad x=0$

$\boxed{c=0}$