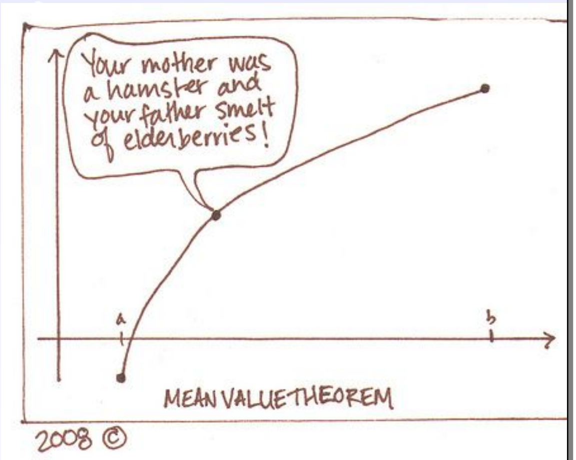
3.2 day 1 - Mean Value Theorem and Rolle's Theorem 1/15/19

## Homework:

- 3.2 A
- 3.1-3.2 PA - Thursday 1/17

**Objective:** Understand and apply the Mean Value Theorem and Rolle's Theorem.




**Do Now:**
Complete the Do Now from the handout on my desk

## Do Now

Find the absolute extrema of the given function on the given interval.
$f(x) = 2\cos(2x)+1$, $[0, 2\pi]$

$f'(x) = -4\sin(2x) = 0$

$2x = 0 + \pi n$

$x = \frac{\pi}{2} n$, n is int.

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$f(0) = 3$

$f(\pi/2) = -1$

$f(\pi) = 3$

$f(3\pi/2) = -1$

$f(2\pi) = 3$

*HW Questions?*

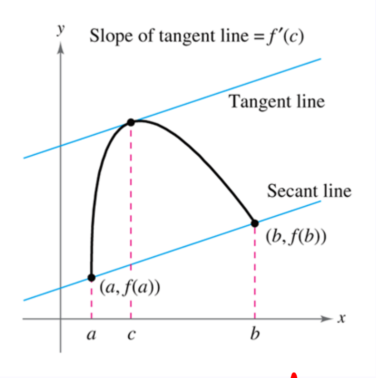
Abs max: $(0,3)$, $(\pi,3)$, $(2\pi,3)$

Abs min: $(\pi/2,-1)$, $(3\pi/2,-1)$

**THEOREM 3.4 The Mean Value Theorem**

If $f$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$, then there exists a number $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$




**Rate of Change (Velocity) Interpretation:**

What does the "mean" refer to in Mean Value Theorem?

Average

**Graphical Interpretation:**

c on (a,b) where tangent line is // secant line between a & b.

NOTE: You must state that the function is continuous on closed interval and differentiable on open interval BEFORE applying theorem.

## Example 1 - Mean Value Theorem

Given $f(x) = 5 - \frac{4}{x}$, find the value of $c$ in the open interval $(1,4)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ if the Mean Value Theorem applies. $f(x) = 5 - 4x^{-1}$

f(x) is everywhere cont & diff except at x=0.

f(x) is cont [1,4] & diff on (1,4).

MVT applies.

$f(1) = 1$

$f(4) = 4$

$f'(x) = \frac{4}{x^2}$

$f'(c) = \frac{f(4) - f(1)}{4-1}$

$\frac{4}{c^2} = \frac{4-1}{3}$

$\frac{4}{c^2} = 1$

$c^2 = 4$

$c = \pm 2$

$c = -2$ not in int.

$\boxed{c = 2}$

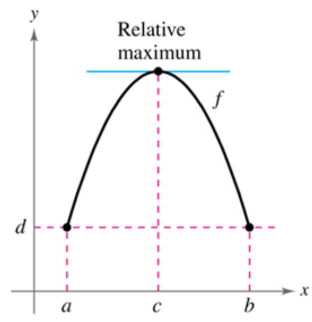
https://www.desmos.com/calculator/qnzk68wrlv

**THEOREM 3.3 Rolle's Theorem**

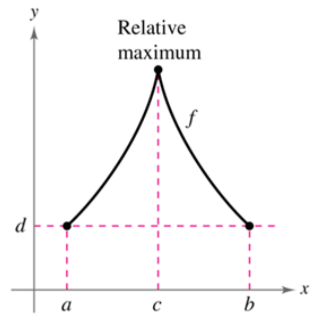
Let $f$ be continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$. If

$$f(a) = f(b)$$

then there is at least one number $c$ in $(a, b)$ such that $f'(c) = 0$.




(a) $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$.




(b) $f$ is continuous on $[a, b]$.

**Rolle's Theorem** states that on a differentiable curve between two values of the same height, there must be a least one point in between where the tangent line is horizontal.

## Example 2 - Rolle's Theorem

Determine whether Rolle's Theorem can be applied to $f(x) = x^3 - 6x^2 + 11x - 2$ on the interval $[1, 3]$. If it can be applied, find all values of $c$ on the interval such that $f'(c) = 0$. If it cannot be applied state why.

f(x) is a polynomial function, ∴ f(x) is cont on [1,3] & diff on (1,3).

$f(1) = 4$

$f(3) = 4$

$f(1) = f(3)$

Rolle's Thm applies.

$f'(x) = 3x^2 - 12x + 11$

$$\boxed{c = \frac{6 \pm \sqrt{3}}{3}}$$

https://www.desmos.com/calculator/qnzk68wrlv

## Example 3

Determine whether Rolle's Theorem can be applied to $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$. If it can be applied, find the value of $c$ that satisfies the theorem. If it cannot be applied state why.

$f(x)$ is cont on $[-2,2]$ & diff on $(-2,2)$.

$f(-2) = 8$

$f(2) = 8$

Rolle's Thm applies.

$f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$

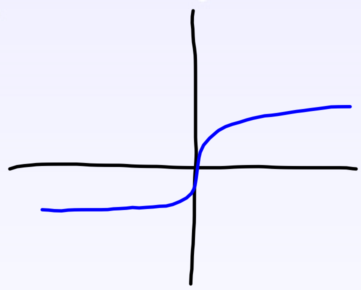
$x = 0, \pm 1$

$\boxed{c = 0, \pm 1}$

## Example 4

Determine whether Mean Value Theorem can be applied to $f(x) = x^{\frac{1}{3}}$ on the interval $[-1, 1]$. If it can be applied, find the value of $c$ that satisfies the theorem. If it cannot be applied state why.

$f(x)$ cont on $[-1,1]$.



$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(x) = \frac{1}{3x^{2/3}}$

$f'(0)$ und $\Rightarrow$ $f(x)$ is non diff at $x=0$, on $(-1,1)$. MVT does NOT apply.

## Example 5

Determine whether Rolle's Theorem can be applied to $f(x)=\dfrac{x^2-1}{x}$ on the interval $[-2,2]$. If it can be applied, find all values of $c$ on the interval such that $f'(c)=0$. If it cannot be applied state why.

## Example 6

Determine whether Rolle's Theorem can be applied to $f(x)=(x-3)(x+1)^2$ on the interval $[-1,3]$. If it can be applied, find all values of $c$ on the interval such that $f'(c)=0$. If it cannot be applied state why.

## Closure

You are driving in a car traveling at 50mph and you pass a police car. You don't worry because you are going the speed limit of 50 mph. Ten minutes later, you pass a second police car and you are traveling at 50mph. The distance between the two police cars is twelve miles. The second police car nails you for speeding. Explain how he can use the Mean Value Theorem to prove that you were speeding in complete sentences.