Name Answer Key   Date________

Calc I H – 3.2 day 2   Period ________

Comparison of the Extreme Value Theorem, Rolle's Theorem, & Mean Value Theorem

Extreme Value Theorem	Rolle's Theorem	Mean Value Theorem
If: • Function is continuous on $[a,b]$	**If:** • Function is continuous on $[a,b]$ • Function is differentiable on (a,b) • $f(a)=f(b)$	**If:** • Function is continuous on $[a,b]$ • Function is differentiable on (a,b)
Then: The function **must** have a maximum **and** a minimum in $[a,b]$.	**Then:** $f'(c)=0$ where c **must** be in the interval (a,b).	**Then:** $f'(c)=\frac{f(b)-f(a)}{b-a}$ where c **must** be in the interval (a,b).
Remember: This is where we evaluate the function values at the **endpoints** and **critical numbers** in order to find the extrema (minimum and maximum).	**Remember:** Maximum and minimum values occur at turning points and $f'(x)=0$ is a horizontal tangent line.	**Remember:** • $\frac{f(b)-f(a)}{b-a}$ is the slope of the **secant line.** • This means the **tangent slope** at point $(c,f(c))$ is the same as the secant slope. • The tangent line $(c,f(c))$ and secant through $(a,f(a))$ and $(b,f(b))$ are parallel.

Important Facts to Remember:

- If c is a critical point, then $f(c)$ must be defined.
- We find critical points because a maximum or a minimum may occur at those values.
- Sharp turns are critical points.
- End points are critical points.
- If $x = c$ is a vertical asymptote, then c is **NOT** a critical point.
- Continuity means the graph has:
  - NO HOLES
  - NO GAPS OR BREAKS
  - NO VERTICAL ASYMPTOTES
- Differentiability means there are **NO SHARP TURNS or VERTICAL TANGENT LINES!**

Guidelines for Finding Extrema on a Closed Interval

1. Find the critical point(s) on $f(x)$ in (a,b)
   a. Find $f'(x)$
   b. Set $f'(x)=0$ and solve.
   c. Find if $f'(x)$ is undefined at any value(s).
   d. Don't forget to check and make sure you only use critical points that are inside the interval.
2. Evaluate the function at the endpoints of the interval and each critical point by substitution into the original function. *(Hint: Use the **TableAsk** or **YVars** feature on your calculator to help with this)*
3. Compare the values that you found in step 2. The least of these values will be the minimum. The greatest of these values will be the maximum.
4. Verify by graphing and using the maximum and minimum feature on your calculator.

Examples:

1. a) Find the extrema of $f(x)=2x^3-54x$ on $[0,4]$.

$f'(x)=6x^2-54$

$f'(x)=0$

$\frac{6x^2-54=0}{6}$

$x^2-9=0$

$x=\pm 3$

$f'(x)$ und

none

$f(0)=0$ Abs max

$f(3)=-108$ Abs min

$f(4)=-88$ NOT extrema

Abs max $(0,0)$ Abs min $(3,-108)$

b) Find the extrema of $g(t)=\dfrac{t^2}{t^2+3}$ on $[-1,1]$.

$g'(t)=\dfrac{2t(t^2+3)-2t(t^2)}{(t^2+3)^2}=\dfrac{6t}{(t^2+3)^2}$

$g'(t)=0$

$6t=0$

$t=0$

$g'(t)$ und

none

$g(-1)=\frac{1}{4}$ abs max

$g(0)=0$ abs min

$g(1)=\frac{1}{4}$ abs max

$(-1,\frac{1}{4}),(1,\frac{1}{4})$ abs max $(0,0)$ abs min

2. Verify that Rolle's Theorem can be applied and then use it to find all values of *c* that satisfy the Theorem given $g(x)=\cos x$ on $[0,2\pi]$.

$g(x)=\cos x$ is cont on $[0,2\pi]$ and diff on $(0,2\pi)$. $g(0)=1=g(2\pi)$

Rolle's thm applies.

$g'(x)=\sin x=0$

$x=\cancel{0},\pi,\cancel{2\pi}$

endpts

$\boxed{c=\pi}$

3. Verify that the Mean Value Theorem can be applied and then use it to find all values of *c* that satisfy the Theorem given $h(x)=\dfrac{1}{x}$ on $[-4,-2]$.

$h(x)$ is everywhere cont & diff everywhere except $x=0$. $h(x)$ is cont on $[-4,-2]$ & diff on $(-4,-2)$. MVT applies.

$h(-4)=-\frac{1}{4}$

$h(-2)=-\frac{1}{2}$

$h'(x)=-\frac{1}{x^2}$

$-\frac{1}{c^2}=\dfrac{-\frac{1}{2}-(-\frac{1}{4})}{-2-(-4)}$

$-\frac{1}{c^2}=\dfrac{-\frac{1}{4}}{2}$

$-\frac{1}{c^2}=-\frac{1}{8}$

$c^2=8$

$c=\pm\sqrt{8}=\pm 2\sqrt{2}$

$\cancel{c=2\sqrt{2}}$ not on $(-4,-2)$

$\boxed{c=-2\sqrt{2}}$