3.2 day 2 Classwork

1. Since $f(x)$ is an even function, $f(a)=f(-a)$ for all values of $a>0$. Since $f(x)$ is a differentiable function on $(-a,a)$, and therefore Continuous, Rolle's theorem holds for all a,

2. $S(t)=200\left(5-\frac{9}{2+t}\right)$ $t=0$, Jan 1, t in months

   a) avg rate of change: $\frac{S(12)-S(0)}{12-0}=\frac{871-100}{12}$

   $= 64.25 \ \frac{\text{units}}{\text{mo.}}$

   b) $S'(t)=200\left(\frac{9}{(2+t)^2}\right)=64.25$

   $1800=64.25(2+t)^2$

   $28.02=(2+t)^2$

   $5.29=2+t$

   $t\approx 3.29$ months

   $\Rightarrow$ In the month of April

3. (a) $f(x)=|x|$ Non differentiable of $[-1,1]$

   Non diff at $x=0$

4. Yes. Since R is differentiable, and therefore continuous on $[0,12]$, Rolle's Thm applies

   Since $R(4)=88=R(12)$, there must exist some value c on $[4,12]$ such that $R'(c)=0$.

5. a) $f(x) = 4 - |x-2|$ on $[-2, 2]$

$$f(x) = \begin{cases} 4+(x-2) & x < -2 \\ 4-(x-2) & x \geq -2 \end{cases}$$

$\therefore$ $f(x)$ is continuous on $[-2,2]$ and diff on $(-2,2)$. However, there are no points a, b in the interval such that $f(a) = f(b)$. Cannot apply Rolle's Thm.

b) $f(x) = \cos(2x)$ on $[\pi/3, 2\pi/3]$

$f(x)$ is everywhere continuous & differentiable

$f(\pi/3) = \cos(2\pi/3) = -\frac{1}{2}$ $\quad f(2\pi/3) = \cos(4\pi/3) = -\frac{1}{2}$

Since $f(\pi/3) = f(2\pi/3)$, Rolle's Thm applies.

$f'(c) = -2\sin(2c) = 0$ $\quad 2c = \pi$

$c = \pi/2$

$\sin(2c) = 0$

6. a) $f(x) = \sqrt{x-3}$ on $[3, 7]$

$f(x)$ cont on $[3,7]$ and differentiable on $(3,7)$

$f'(x) = \dfrac{1}{2\sqrt{x-3}}$ $\quad f(7) = 2 \quad f(3) = 0$

$\dfrac{f(7) - f(3)}{7-3} = \dfrac{2-0}{4} = \dfrac{1}{2}$

$\dfrac{1}{2} = \dfrac{1}{2\sqrt{c-3}}$ $\quad \sqrt{c-3} = 1$

$c - 3 = 1$

$c = 4$

3.2 day 2 Cont.

6. b) $f(x) = 2\cos x + \cos(2x)$ on $[0, \pi]$

$f(x)$ Cont on $[0,\pi]$ and diff on $(0,\pi)$.

$f'(x) = -2\sin x - 2\sin(2x)$ $\quad f(0) = 2+1 = 3$

$f(\pi) = -2+1 = -1$

$$\frac{f(\pi)-f(0)}{\pi - 0} = \frac{-1-3}{\pi} = \frac{-4}{\pi} = -2(\sin c + \sin(2c))$$

$$\sin c + \sin(2c) = \frac{2}{\pi}$$

On GC, $c = .2173$

7. $f(x) = x^2 + \sin x + 4x$ on $[-2, 2]$

$f(x)$ Cont on $[-2,2]$ and diff on $(-2,2)$.

a) $f'(x) = 2x + \cos x + 4$ $\quad f(-2) = 4 + \sin(-2) - 8$

$= \sin(-2) - 4$ ✓

$f(2) = 4 + \sin(2) + 8$

$= \sin(2) + 12$ ✓

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{\sin(2) + 12 - (\sin(-2) - 4)}{4}$$

Since $\sin x$ is odd function $\sin(-2) = -\sin(2)$

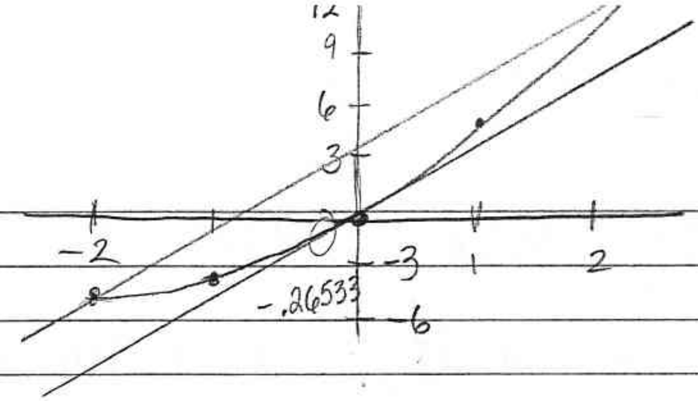
$$\frac{f(2)-f(-2)}{4} = \frac{\sin(2) + \sin(2) + 16}{4} = \frac{2\sin(2) + 16}{4}$$

$$= \frac{\sin(2) + 8}{2}$$

b) $f'(c) = 2c + \cos c + 4 = \dfrac{\sin(2) + 8}{2}$

$c \approx -.2563$ $\qquad m = 4.4547$ ✓

c)



8. $f(x) = x + \sin x$ on $[0, b]$

$f(b) = b + \sin b$ $\qquad \dfrac{f(b)-f(0)}{b-0} = \dfrac{b+\sin b}{b} = m_{sec}$

$f(0) = 0$

$f'(c) = 1 + \cos c = \dfrac{b + \sin b}{b} = 1 + \dfrac{\sin b}{b}$

$\cos c = \dfrac{\sin b}{b} \Rightarrow \sin b - b\cos(c) = 0$

for $c = 4.6658$ | for $c = 1.6174$

$\sin b - b\cos(4.6658) = 0$ | $\sin b - b\cos(1.6174) =$

$b = 6$ | $b = 6$

9. $v(t)$ diff function on $[0, 30]$, $\therefore$ Cont.

Yes. For example, $\dfrac{v(25) - v(20)}{25-20} = \dfrac{0.4 - 0.5}{5}$

$= \dfrac{-.1}{5} = -.02 \dfrac{m}{min^2}$

By the MVT, there exists a time c in $(20, 25)$

Such that $a(t) = v'(t) = -.02 \dfrac{m}{min^2}$

Since $v(t) > 0$ at this time, but $a(t) < 0$, the cyclist is slowing down.