

Name _____

Date _____

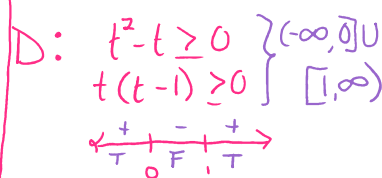
Calc I H - 3.3 - 3.4 Review

Period _____

1. Find any critical numbers of the function $g(t) = \sqrt{t^2 - t} = (t^2 - t)^{1/2}$

$$g'(t) = \frac{1}{2\sqrt{t^2 - t}} \cdot 2t - 1 = \frac{2t - 1}{2\sqrt{t^2 - t}}$$

$g'(t) = 0 \Rightarrow t = \frac{1}{2}$ (NOT in domain)
 $g'(t)$ und $t^2 - t = 0$
 $t(t - 1) = 0 \Rightarrow t = 0, 1$



For #2-3 complete the following:

- Find the critical numbers of f (if any);
- Find the open intervals where the function is increasing or decreasing (Justify);
- Apply the First Derivative Test to identify all relative extrema (Justify);
- Find the open intervals where the function is concave up or down (Justify); and
- Identify all Points of Inflection (Justify).

2. For the function $f(x) = 4x^3 - 18x^2 + 3$:

$$f'(x) = 12x^2 - 36x$$

$$f'(x) = 0 \Rightarrow 12x(x - 3) = 0$$

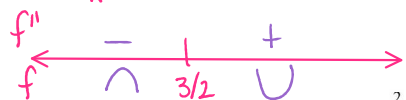
$f'(x)$ und None

Crit #: $x = 0, 3$

$$f''(x) = 24x - 36$$

$$f''(x) = 0 \Rightarrow 12(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$$

$f''(x)$ und None



3. For the function $f(x) = (x - 1)^{2/3}$:

$$f'(x) = \frac{2}{3(x - 1)^{1/3}}$$

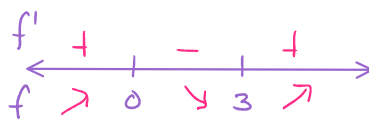
$$f'(x) = 0 \Rightarrow \text{None}$$

$f'(x)$ und $x = 1$

$$f''(x) = \frac{-2}{9(x - 1)^{4/3}}$$

$$f''(x) = 0 \Rightarrow \text{None}$$

$f''(x)$ und $x = 1$



Incr: $(-\infty, 0) \cup (3, \infty)$ $f' > 0$

Decr: $(0, 3)$ $f' < 0$

Rel max: $(0, 3)$ $f' > 0 \rightarrow f' < 0$

Rel min: $(3, -5)$ $f' < 0 \rightarrow f' > 0$

Conc up: $(\frac{3}{2}, \infty)$, $f'' > 0$

Conc down: $(-\infty, \frac{3}{2})$, $f'' < 0$

POI: $(\frac{3}{2}, -24)$ f'' changes sign



Incr: $(1, \infty)$ $f' > 0$

Decr: $(-\infty, 1)$ $f' < 0$

Rel min: $(1, 0)$ $f' < 0 \rightarrow f' > 0$

Rel max: None



Conc down: $(-\infty, 1) \cup (1, \infty)$ $f'' < 0$

Conc up: None

POI: None

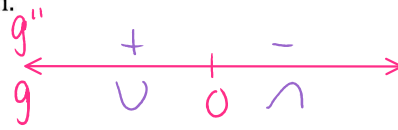
4. Determine the open intervals on which the graph of $g(x) = -7x^3 - 3x - 5$ is concave downward or concave upward and identify all points of inflection.

$$g'(x) = -21x^2 - 3$$

$$g''(x) = -42x$$

$$\frac{g''(x) = 0}{x = 0}$$

$g''(x)$ und
none



Conc Up: $(-\infty, 0)$ $g'' > 0$

Conc down: $(0, \infty)$ $g'' < 0$

POI: $(0, -5)$ g'' changes sign

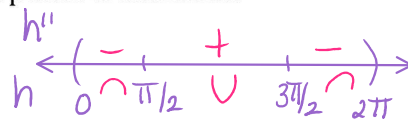
5. Determine the open intervals on which the graph of $h(x) = x + \cos x$ is concave downward or concave upward on the interval $(0, 2\pi)$ and identify all points of inflection.

$$h'(x) = 1 - \sin x$$

$$h''(x) = -\cos x = 0$$

$$\frac{h''(x) = 0}{x = \pi/2, 3\pi/2}$$

h'' und
none



Conc Up: $(\pi/2, 3\pi/2)$ $h'' > 0$

Conc down: $(0, \pi/2) \cup (3\pi/2, 2\pi)$ $h'' < 0$

POI: $(\pi/2, \pi/2)$ & $(3\pi/2, 3\pi/2)$
 h'' changes sign

6. Use the Second Derivative Test, where applicable, to find all relative extrema of the function $f(x) = x^3 + 3x^2 - 8$.

$$f'(x) = 3x^2 + 6x = 0$$

$$3x(x+2) = 0$$

Crit #'s: $x = 0, -2$

$$f''(x) = 6x + 6$$

$$f''(0) = 6 > 0 \text{ \& } f'(0) = 0 \quad \cup$$

$(0, -8)$ Rel min

$$f''(-2) = -6 < 0 \text{ \& } f'(-2) = 0 \quad \cap$$

$(-2, -4)$ Rel max

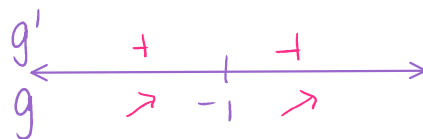
7. Find the critical numbers (if any) and the open intervals on which the function is increasing or decreasing for $g(x) = (x+1)^3$.

$$g'(x) = 3(x+1)^2$$

$$\frac{g'(x) = 0}{3(x+1)^2 = 0}$$

Crit #: $x = -1$

$g'(x)$ und
none



Incr: $(-\infty, -1) \cup (-1, \infty)$ $f' > 0$

Decr: None

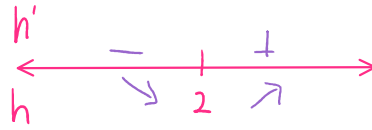
No extrema!

8. Use the First Derivative Test to find any relative extrema of the function $h(t) = \frac{1}{4}t^4 - 8t$.

$$h'(t) = t^3 - 8 = 0$$

$$t^3 = 8$$

$$t = 2$$



Rel min: $(2, -12)$ $h' < 0 \rightarrow h' > 0$

9. Determine the point(s) of inflection and discuss the concavity of the graph for each function:

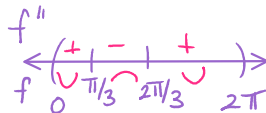
a. $f(x) = \frac{\sqrt{3}}{4}x^2 + \sin x$ on the interval $(0, 2\pi)$.

$$f' = \frac{\sqrt{3}}{2}x + \cos x$$

$$f'' = \frac{\sqrt{3}}{2} - \sin x = 0$$

$$\frac{f'' = 0}{\sin x = \frac{\sqrt{3}}{2}} \quad \begin{array}{l} f'' \text{ und} \\ \text{none} \end{array}$$

$$x = \pi/3, 2\pi/3$$



Conc up: $(0, \pi/3) \cup (2\pi/3, \infty)$ $f'' > 0$

Conc down: $(\pi/3, 2\pi/3)$ $f'' < 0$

POI: $(\pi/3, 1.341)$, $(2\pi/3, 2.765)$ f'' changes sign

b. $f(x) = (x+2)^2(x-4)$

$$f'(x) = 2(x+2)(x-4) + (x+2)^2$$

$$f'(x) = (x+2)(2x-8+x+2)$$

$$f'(x) = (x+2)(3x-6)$$

$$f''(x) = 3x-6 + 3(x+2) = 6x$$

$$\frac{f''(x) = 0}{6x = 0}$$

$$x = 0$$

f'' und
none



Conc up: $(0, \infty)$ $f'' > 0$

Conc down: $(-\infty, 0)$ $f'' < 0$

POI: $(0, -16)$ f'' changes sign

10. Use the Second Derivative Test, where applicable, to find all relative extrema of the function $f(x) = 6x^5 - 10x^3$.

$$f'(x) = 30x^4 - 30x^2 = 0$$

$$30x^2(x^2 - 1) = 0$$

$$x = 0, \pm 1$$

$$f''(-1) = -60 < 0,$$

$$f'(-1) = 0$$

Rel max $(-1, 4)$

$$f''(x) = 120x^3 - 60x$$

$$f''(0) = 0$$

Inconclusive

f'



NOT extrema

$$f''(1) = 60 > 0,$$

$$f'(1) = 0$$

Rel min $(1, -4)$