Name________________________________ **Date**____________

Calc I H – 3.3 – 3.4 Review **Period** __________

1. Find any critical numbers of the function $g(t)=\sqrt{t^2-t}$. $=(t^2-t)^{1/2}$

$g'(t)=\dfrac{1}{2\sqrt{t^2-t}}\cdot 2t-1=\dfrac{2t-1}{2\sqrt{2t^2-t}}$

$g'(t)=0$: $\cancel{t=1/2}$ NOT in domain

$g'(t)$ und: $t^2-t=0$, $t(t-1)=0$, $\boxed{t=0,1}$

D: $t^2-t\ge 0$, $t(t-1)\ge 0$; $(-\infty,0]\cup[1,\infty)$

$+$ T | 0 | $-$ F | 1 | $+$ T

For #2-3 complete the following:

a. Find the critical numbers of f (if any);
b. Find the open intervals where the function is increasing or decreasing (Justify);
c. Apply the First Derivative Test to identify all relative extrema (Justify);
d. Find the open intervals where the function is concave up or down (Justify); and
e. Identify all Points of Inflection (Justify).

2. For the function $f(x)=4x^3-18x^2+3$:

$f'(x)=12x^2-36x$

$f'(x)=0$: $12x(x-3)=0$ $\quad$ $f'(x)$ und: None

$\boxed{\text{Crit\#: } x=0,3}$

f': $+$ | 0 | $-$ | 3 | $+$
f: $\nearrow$ | 0 | $\searrow$ | 3 | $\nearrow$

Incr: $(-\infty,0)\cup(3,\infty)$ $f'>0$
Decr: $(0,3)$ $f'<0$
Rel max: $(0,3)$ $f'>0\to f'<0$
Rel min: $(3,-51)$ $f'<0\to f'>0$

$f''(x)=24x-36$

$f''(x)=0$: $12(2x-3)=0$, $x=3/2$ $\quad$ $f''(x)$ und: None

f'': $-$ | 3/2 | $+$
f: $\cap$ | 3/2 | $\cup$

Conc up: $(3/2,\infty)$, $f''>0$
Conc down: $(-\infty,3/2)$, $f''<0$
POI: $(3/2,-24)$ f'' changes sign

3. For the function $f(x)=(x-1)^{\frac{2}{3}}$:

$f'(x)=\dfrac{2}{3(x-1)^{1/3}}$

$f'(x)=0$: None $\quad$ $f'(x)$ und: $3(x-1)^{1/3}=0$, $x=1$

f': $-$ | 1 | $+$
f: $\searrow$ | 1 | $\nearrow$

Incr: $(1,\infty)$ $f'>0$
Decr: $(-\infty,1)$ $f'<0$
Rel min: $(1,0)$ $f'<0\to f'>0$
Rel max: None

$f''(x)=\dfrac{-2}{9(x-1)^{4/3}}$

$f''(x)=0$: None $\quad$ $f''(x)$ und: $9(x-1)^{4/3}=0$, $x=1$

f'': $-$ | 1 | $-$
f: $\cap$ | 1 | $\cap$

Conc down: $(-\infty,1)\cup(1,\infty)$ $f''<0$
Conc up: None $\quad$ POI: None

4. Determine the open intervals on which the graph of $g(x) = -7x^3 - 3x - 5$ is concave downward or concave upward and identify all points of inflection.

$g'(x) = -21x^2 - 3$

$g''(x) = -42x$

$g''(x) = 0$: $x = 0$

$g''(x)$ und: none

g'': $+$ | 0 | $-$

g: $\cup$ | 0 | $\cap$

Conc Up: $(-\infty, 0)$ $g'' > 0$

Conc down: $(0, \infty)$ $g'' < 0$

POI: $(0, -5)$ g'' changes sign

5. Determine the open intervals on which the graph of $h(x) = x + \cos x$ is concave downward or concave upward on the interval $(0, 2\pi)$ and identify all points of inflection.

$h'(x) = 1 - \sin x$

$h''(x) = -\cos x = 0$

$h''(x) = 0$: $x = \pi/2, 3\pi/2$

h'' und: none

h'': ($-$ | $+$ | $-$)

h: 0 $\cap$ $\pi/2$ $\cup$ $3\pi/2$ $\cap$ 2π

Conc Up: $(\pi/2, 3\pi/2)$ $h'' > 0$

Conc down: $(0, \pi/2) \cup (3\pi/2, 2\pi)$ $h'' < 0$

POI: $(\pi/2, \pi/2)$ & $(3\pi/2, 3\pi/2)$

h'' changes sign

6. Use the Second Derivative Test, where applicable, to find all relative extrema of the function $f(x) = x^3 + 3x^2 - 8$.

$f'(x) = 3x^2 + 6x = 0$

$3x(x+2) = 0$

Crit #'s: $x = 0, -2$

$f''(x) = 6x + 6$

$f''(0) = 6 > 0$ & $f'(0) = 0$ $\cup$

$(0, -8)$ Rel min

$f''(-2) = -6 < 0$ & $f'(-2) = 0$ $\cap$

$(-2, -4)$ Rel max

7. Find the critical numbers (if any) and the open intervals on which the function is increasing or decreasing for $g(x) = (x+1)^3$.

$g'(x) = 3(x+1)^2$

$g'(x) = 0$

$3(x+1)^2 = 0$

Crit #: $x = -1$

$g'(x)$ und: none

g': $+$ | -1 | $+$

g: $\nearrow$ | -1 | $\nearrow$

Incr: $(-\infty, -1) \cup (-1, \infty)$ $f' > 0$

Decr: None

No extrema!

8. Use the First Derivative Test to find any relative extrema of the function $h(t) = \frac{1}{4}t^4 - 8t$.

$h'(t) = t^3 - 8 = 0$

$t^3 = 8$

$t = 2$

h': $-$ | $+$

h: $\searrow$ 2 $\nearrow$

Rel min: $(2, -12)$ $h' < 0 \to h' > 0$

9. Determine the point(s) of inflection and discuss the concavity of the graph for each function:

a. $f(x) = \frac{\sqrt{3}}{4}x^2 + \sin x$ on the interval $(0, 2\pi)$.	b. $f(x) = (x+2)^2(x-4)$
$f' = \frac{\sqrt{3}}{2}x + \cos x$ $f'' = \frac{\sqrt{3}}{2} - \sin x = 0$ $f'' = 0$: $\sin x = \frac{\sqrt{3}}{2}$; f'' und: none $x = \pi/3,\ 2\pi/3$ f'': $+$ | $-$ | $+$ f: 0 $\cup$ $\pi/3$ $\cap$ $2\pi/3$ $\cup$ 2π Conc up: $(0, \pi/3) \cup (2\pi/3, \infty)$ $f'' > 0$ Conc down: $(\pi/3, 2\pi/3)$ $f'' < 0$ POI: $(\pi/3, 1.341)$, $(2\pi/3, 2.765)$ f'' changes sign	$f'(x) = 2(x+2)(x-4) + (x+2)^2$ $f'(x) = (x+2)(2x-8+x+2)$ $f'(x) = (x+2)(3x-6)$ $f''(x) = 3x-6 + 3(x+2) = 6x$ $f''(x) = 0$: $6x = 0$, $x = 0$; f'' und: none f'': $-$ | $+$ f: $\cap$ 0 $\cup$ Conc up: $(0, \infty)$ $f'' > 0$ Conc down: $(-\infty, 0)$ $f'' < 0$ POI: $(0, -16)$ f'' changes sign

10. Use the Second Derivative Test, where applicable, to find all relative extrema of the function $f(x) = 6x^5 - 10x^3$.

$f'(x) = 30x^4 - 30x^2 = 0$

$30x^2(x^2 - 1) = 0$

$x = 0, \pm 1$

$f''(x) = 120x^3 - 60x$

$f''(-1) = -60 < 0$, $f'(-1) = 0$

Rel max $(-1, 4)$

$f''(0) = 0$

Inconclusive

f': $-$ | $-$

f: -1 $\searrow$ 0 $\searrow$ 1

NOT extrema

$f''(1) = 60 > 0$, $f'(1) = 0$

Rel min $(1, -4)$