

Name _____

Date _____

Calc I H - 3.3 day 3 - 1st Derivative Test

Period _____

Directions:

For each given function find:

- a) the critical numbers of f (if any exist).
- b) the open interval(s) on which the function is increasing or decreasing.
- c) any relative extrema (if they exist) using the First Derivative Test.

1. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$
 $f'(x) = 0$ $f'(x)$ und
 $3x(x-4) = 0$ none
 $x = 0, 4$

$f'(x)$ + - +
 $f(x)$ ↗ 0 ↘ 4 ↗

Incr: $(-\infty, 0) \cup (4, \infty)$ $f'(x) > 0$
Decr: $(0, 4)$ $f'(x) < 0$
Rel max: $(0, 15)$ $f' > 0 \rightarrow f' < 0$
Rel min: $(4, -17)$ $f' < 0 \rightarrow f' > 0$

2. $g(x) = x^{\frac{2}{3}} - 4$

$g'(x) = \frac{2}{3x^{1/3}}$
 $g'(x) = 0$ $g'(x)$ und
 none $x = 0$

$g'(x)$ - +
 $g(x)$ ↘ 0 ↗

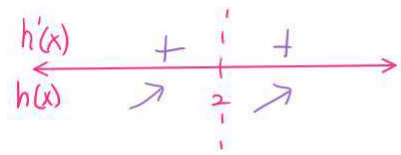
Decr: $(-\infty, 0)$ $g' < 0$
Incr: $(0, \infty)$ $g' > 0$
Rel max: none
Rel min: $(0, -4)$ $g' < 0 \rightarrow g' > 0$

3. $h(x) = \frac{x^2 - 3x - 4}{x - 2}$

$h'(x) = \frac{(2x-3)(x-2) - (x^2-3x-4)}{(x-2)^2}$
 $= \frac{2x^2 - 7x + 6 - x^2 + 3x + 4}{(x-2)^2}$
 $= \frac{x^2 - 4x + 10}{(x-2)^2}$

$h'(x) = 0$
 $x^2 - 4x + 10 = 0$
 $x^2 - 4x + 4 = -10 + 4$ $(x-2)^2 \neq -6$
 NOT possible!
 No Soln

$h'(x)$ und
 $(x-2)^2 = 0$
 $x = 2$



Note: VA at $x = 2$, NOT Crit #
Incr: $(-\infty, 2) \cup (2, \infty)$ $h'(x) > 0$
Decr:
Rel max:
Rel min: } none

4. $p(x) = x + 2\sin x$ on the interval $(0, 2\pi)$.

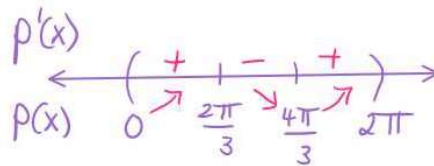
$$p'(x) = 1 + 2\cos x$$

$$\frac{p'(x) = 0}{1 + 2\cos x = 0}$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$\frac{p'(x)}{\text{none}}$



Incr: $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$ $p'(x) > 0$

Decr: $(\frac{2\pi}{3}, \frac{4\pi}{3})$ $p'(x) < 0$

Rel max: $(\frac{2\pi}{3}, 3.826)$ $p'(x) > 0 \rightarrow p'(x) < 0$
 $\frac{2\pi}{3} + \sqrt{3}$

Rel min: $(\frac{4\pi}{3}, 2.451)$ $p'(x) < 0 \rightarrow p'(x) > 0$
 $\frac{4\pi}{3} - \sqrt{3}$

5. $r(x) = -e^{\frac{1}{2}x^4 - x^2} - 3$

$$r'(x) = -(2x^3 - 2x)e^{\frac{1}{2}x^4 - x^2}$$

$$\frac{r'(x) = 0}{-(2x^3 - 2x)e^{\frac{1}{2}x^4 - x^2} = 0}$$

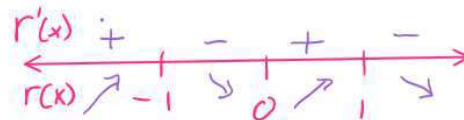
$$e^{\frac{1}{2}x^4 - x^2} \neq 0$$

$$2x^3 - 2x = 0$$

$$2x(x^2 - 1) = 0$$

$$x = 0, \pm 1$$

$\frac{r'(x)}{\text{none}}$



Incr: $(-\infty, -1) \cup (0, 1)$ $r'(x) > 0$

Decr: $(-1, 0) \cup (1, \infty)$ $r'(x) < 0$

Rel max: $(-1, -e^{-\frac{1}{2}} - 3)$, $(1, -e^{-\frac{1}{2}} - 3)$
 -3.607

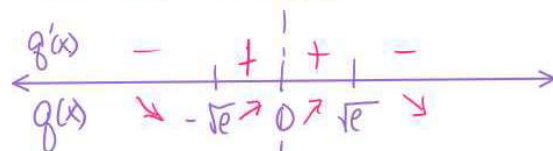
Rel min: $(0, -4)$ $r'(x) < 0 \rightarrow r'(x) > 0$

6. $q(x) = \frac{\ln(x^2) + 1}{x}$

$$q'(x) = \frac{\frac{1}{x^2}(2x)(x) - (\ln x^2 + 1)}{x^2}$$

$$= \frac{1 - \ln x^2}{x^2}$$

NOTE: Domain $(-\infty, 0) \cup (0, \infty)$



Incr: $(-\sqrt{e}, 0) \cup (0, \sqrt{e})$ $q'(x) > 0$

Decr: $(-\infty, -\sqrt{e}) \cup (\sqrt{e}, \infty)$ $q'(x) < 0$

Rel max: $(\sqrt{e}, \frac{2}{\sqrt{e}})$ $q'(x) > 0 \rightarrow q'(x) < 0$
1.213

Rel min: $(-\sqrt{e}, -\frac{2}{\sqrt{e}})$ $q'(x) < 0 \rightarrow q'(x) > 0$
-1.213

$$\frac{q'(x) = 0}{1 - \ln x^2 = 0}$$

$$\ln x^2 = 1$$

$$x^2 = e$$

$$x = \pm\sqrt{e}$$

$\frac{q'(x)}{x=0}$