Name____________________________________ **Date**______________

Calc I H – 3.3 day 3 – 1st Derivative Test **Period** _____________

Directions:
For each given function find:

a) the critical numbers of *f* (if any exist).
b) the open interval(s) on which the function is increasing or decreasing.
c) any relative extrema (if they exist) using the First Derivative Test.

1. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$f'(x) = 0$

$3x(x-4) = 0$

$x = 0, 4$

$f'(x)$ und

none

$f'(x)$: $+$ | $-$ | $+$

$f(x)$: $\nearrow$ 0 $\searrow$ 4 $\nearrow$

Incr: $(-\infty, 0) \cup (4, \infty)$ $f'(x) > 0$

Decr: $(0, 4)$ $f'(x) < 0$

Rel max: $(0, 15)$ $f' > 0 \rightarrow f' < 0$

Rel min: $(4, -17)$ $f' < 0 \rightarrow f' > 0$

2. $g(x) = x^{\frac{2}{3}} - 4$

$g'(x) = \dfrac{2}{3x^{1/3}}$

$g'(x) = 0$

none

$g'(x)$ und

$x = 0$

$g'(x)$: $-$ | $+$

$g(x)$: $\searrow$ 0 $\nearrow$

Decr: $(-\infty, 0)$ $g' < 0$

Incr: $(0, \infty)$ $g' > 0$

Rel max: none

Rel min: $(0, -4)$ $g' < 0 \rightarrow g' > 0$

3. $h(x) = \dfrac{x^2 - 3x - 4}{x - 2}$

$h'(x) = \dfrac{(2x-3)(x-2) - (x^2-3x-4)}{(x-2)^2}$

$= \dfrac{2x^2 - 7x + 6 - x^2 + 3x + 4}{(x-2)^2}$

$= \dfrac{x^2 - 4x + 10}{(x-2)^2}$

$h'(x) = 0$

$x^2 - 4x + 10 = 0$

$x^2 - 4x + 4 = -10 + 4$

$(x-2)^2 \neq -6$

Not possible!
No soln

$h'(x)$ und

$(x-2)^2 = 0$

$x = 2$

$h'(x)$: $+$ ⋮ $+$

$h(x)$: $\nearrow$ 2 $\nearrow$

Note: VA at $x = 2$, NOT crit #

Incr: $(-\infty, 2) \cup (2, \infty)$ $h'(x) > 0$

Decr:
Rel max:
Rel min:
} none

4. $p(x) = x + 2\sin x$ on the interval $(0, 2\pi)$.

$p'(x) = 1 + 2\cos x$

$p'(x) = 0$

$1 + 2\cos x = 0$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$p'(x)$ und: none

$p'(x)$: $($ + | − | + $)$

$p(x)$: $0 \nearrow \frac{2\pi}{3} \searrow \frac{4\pi}{3} \nearrow 2\pi$

Incr: $(0, 2\pi/3) \cup (4\pi/3, 2\pi)$ $p'(x) > 0$

Decr: $(2\pi/3, 4\pi/3)$ $p'(x) < 0$

Rel max: $(2\pi/3, \underset{\frac{2\pi}{3}+\sqrt{3}}{3.826})$ $p'(x) > 0 \to p'(x) < 0$

Rel min: $(4\pi/3, \underset{\frac{4\pi}{3}-\sqrt{3}}{2.457})$ $p'(x) < 0 \to p'(x) > 0$

5. $r(x) = -e^{\left(\frac{1}{2}x^4 - x^2\right)} - 3$

$r'(x) = -(2x^3 - 2x)e^{\left(\frac{1}{2}x^4 - x^2\right)}$

$r'(x) = 0$

$-(2x^3 - 2x)e^{\frac{1}{2}x^4 - x^2} = 0$

$e^{\frac{1}{2}x^4 - x^2} \neq 0$

$2x^3 - 2x = 0$

$2x(x^2 - 1) = 0$

$x = 0, \pm 1$

$r'(x)$ und: none

$r'(x)$: + | − | + | −

$r(x)$: $\nearrow -1 \searrow 0 \nearrow 1 \searrow$

Incr: $(-\infty, -1) \cup (0, 1)$ $r'(x) > 0$

Decr: $(-1, 0) \cup (1, \infty)$ $r'(x) < 0$

Rel max: $(-1, \underset{-3.607}{-e^{-\frac{1}{2}} - 3})$, $(1, \underset{-3.607}{-e^{-\frac{1}{2}} - 3})$

$r'(x) > 0 \to r'(x) < 0$

Rel min: $(0, -4)$ $r'(x) < 0 \to r'(x) > 0$

6. $q(x) = \dfrac{\ln(x^2) + 1}{x}$

$q'(x) = \dfrac{\frac{1}{x^2}(2x)(x) - (\ln x^2 + 1)}{x^2}$

$= \dfrac{1 - \ln x^2}{x^2}$

$q'(x) = 0$

$1 - \ln x^2 = 0$

$\ln x^2 = 1$

$x^2 = e$

$x = \pm\sqrt{e}$

$q'(x)$ und: $x = 0$

NOTE: Domain $(-\infty, 0) \cup (0, \infty)$

$q'(x)$: − | + ¦ + | −

$q(x)$: $\searrow -\sqrt{e} \nearrow 0 \nearrow \sqrt{e} \searrow$

Incr: $(-\sqrt{e}, 0) \cup (0, \sqrt{e})$ $q'(x) > 0$

Decr: $(-\infty, -\sqrt{e}) \cup (\sqrt{e}, \infty)$ $q'(x) < 0$

Rel max: $(\sqrt{e}, \underset{1.213}{\frac{2}{\sqrt{e}}})$ $q'(x) > 0 \to q'(x) < 0$

Rel min: $(-\sqrt{e}, \underset{-1.213}{\frac{2}{-\sqrt{e}}})$, $q'(x) < 0 \to q'(x) > 0$