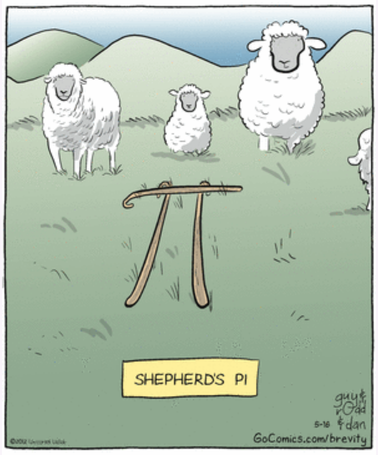
3.3 Day 4 - First Derivative Test Practice 2/4/19

Homework:

- Complete 3.3 Review - due Thursday, 2/7
- Quiz 3.3 Thursday 2/7 - Scientific Calc Only



Objective:

Use the first derivative to find intervals where a function is increasing and decreasing and us the first derivative test to find all extrema.

Do Now:

Find the intervals where the given function is increasing and decreasing.

$$f(x) = 3x(x-3)^{\frac{1}{3}}$$

Do Now

Find the intervals where the given function is increasing and decreasing.

$$f(x) = 3x(x-3)^{\frac{1}{3}} \Rightarrow D: (-\infty, \infty)$$

$f'(x) = 3(x-3)^{1/3} + \cancel{3}x \cdot \frac{1}{\cancel{3}}(x-3)^{-2/3}$

$= 3(x-3)^{1/3} + \frac{x}{(x-3)^{2/3}}$

$f'(x) = 0$

$3(x-3)^{1/3} + \frac{x}{(x-3)^{2/3}} = 0$

$\frac{3(x-3)^{1/3}}{1} = \frac{-x}{(x-3)^{2/3}}$

$3(x-3) = -x$

$3x - 9 = -x$

$4x - 9 = 0$

$x = 9/4 = 2\frac{1}{4}$

$f'(x)$ und

$\left((x-3)^{2/3}\right)^{3/2} = 0^{3/2}$

$x - 3 = 0$

$x = 3$

f': $-$ | $\frac{9}{4}$ | $+$ | $\frac{5}{2}$ | 3 | $+$

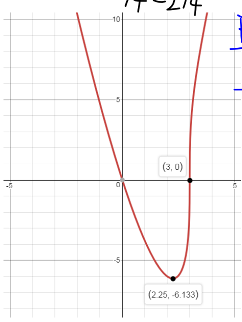
f: $\searrow$ $\frac{9}{4}$ $\nearrow$ 3 $\nearrow$

Incr $(9/4, 3) \cup (3, \infty)$ $f' > 0$

Decr $(-\infty, 9/4)$ $f' < 0$

Rel min $(9/4, \quad)$ $f' < 0 \to f' > 0$

Rel max none



More Fun...

What is the domain of each function?
Which functions have vertical asymptotes?

$$f(x)=\frac{3x+2}{3x-2}$$

D: $(-\infty, 2/3) \cup (2/3, \infty)$

$$g(x)=(2x+5)^{1/2} = \sqrt{2x+5}$$

$2x+5 \geq 0$

$x \geq -\frac{5}{2}$

D: $\left[-\frac{5}{2}, \infty\right)$

$$h(x)=(2x-1)^{1/3} = \sqrt[3]{2x-1}$$

D: $(-\infty, \infty)$

$$j(x)=\frac{x^2-1}{x+1}$$

$j(x)=\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}$

D: $(-\infty, -1) \cup (-1, \infty)$

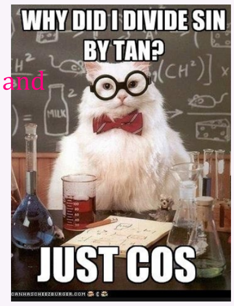
$$k(x)=\frac{5x^2-1}{x^2}$$

$x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

Getting Triggy with it!

Find the intervals where function is increasing and decreasing and locate all relative extrema.



$$y=\cos^2 x-\cos x \quad (0, 2\pi)$$

$y' = 2\cos x(-\sin x) + \sin x$

$= -2\cos x \sin x + \sin x = 0$

$\sin x(-2\cos x + 1) = 0$

$\sin x = 0$ $\quad$ $\cos x = \frac{1}{2}$

$x = \pi$ $\quad$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

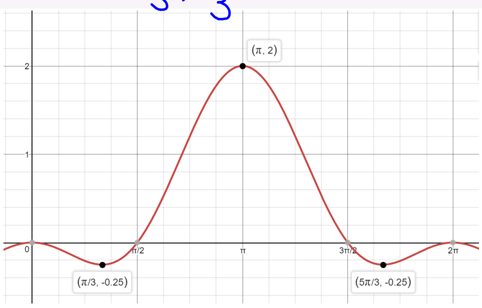
$(-1,0)$ $(0,1)$ $(1,0)$ $(0,-1)$

(x, y)

$(\cos\theta, \sin\theta)$

y': (0 $\pi/3$ π $5\pi/3$ 2π)

y



Get into your groups to begin the review sheet finding intervals where functions are increasing and decreasing and locate all relative extrema.

I can do things you cannot, you can do things I cannot; together we can do great things.

- Mother Teresa

3.3 Review

Sunday, February 18, 2018 5:23 PM

Name Answer Key Date________

Calc I H – 3.3 Review Period ________

For each function, find: the critical numbers *(if any)*, the open interval(s) on which the function is increasing or decreasing, and all relative extrema using the First Derivative Test.

1. $f(x)=(x-1)^2(x+2)$

$f'(x)=2(x-1)(x+2)+(x-1)^2$
$=2(x^2+x-2)+x^2-2x+1$
$=2x^2+\cancel{2x}-4+x^2-\cancel{2x}+1$
$=3x^2-3$

$f'(x)=0$: $3x^2-3=0$, $3(x^2-1)=0$, $x=\pm 1$

~~f'(x) und~~ Not Poss.

$f(-1)=(-2)^2(1)=4$, $f(1)=0$

$f'(x)$: $+$ | -1 | $-$ | 1 | $+$
$f(x)$: $\nearrow$ -1 $\searrow$ 1 $\nearrow$

Incr: $(-\infty,-1)\cup(1,\infty)$, $f'(x)>0$
Decr: $(-1,1)$, $f'(x)<0$
Rel Max: $(-1,4)$, $f(x)$ incr → decr
Rel Min: $(1,0)$, $f(x)$ decr → incr

2. $f(x)=\dfrac{x^2-4x+4}{x+2}$

$f'(x)=\dfrac{(2x-4)(x+2)-(x^2-4x+4)}{(x+2)^2}$
$=\dfrac{2x^2-8-x^2+4x-4}{(x+2)^2}=\dfrac{x^2+4x-12}{(x+2)^2}$

$f'(x)=0$: $x^2+4x-12=0$, $(x+6)(x-2)=0$, $x=-6,2$

$f'(x)$ und: $x+2=0$, $x=-2$ Not in domain

$f'(x)$: $+$ | -6 | $-$ | -2 | $-$ | 2 | $+$
$f(x)$: $\nearrow$ -6 $\searrow$ -2 $\searrow$ 2 $\nearrow$

Incr: $(-\infty,-6)\cup(2,\infty)$, $f'(x)>0$
Decr: $(-6,-2)\cup(-2,2)$, $f'(x)<0$
Rel Max: $(-6,-16)$, $f(x)$ incr → decr
Rel Min: $(2,0)$, $f(x)$ decr → incr

3. $f(x)=x^3+3x^2-45x+17$

$f'(x)=3x^2+6x-45$

$f'(x)=0$: $3x^2+6x-45=0$, $x^2+2x-15=0$, $(x+5)(x-3)=0$, $x=-5,3$

~~f'(x) und~~ Not Poss

$f'(x)$: $+$ | -5 | $-$ | 3 | $+$
$f(x)$: $\searrow$ -5 $\nearrow$ 3 $\searrow$

Incr: $(-\infty,-5)\cup(3,\infty)$, $f'(x)>0$
Decr: $(-5,3)$, $f'(x)<0$
Rel Max: $(-5,102)$ $f(x)$ incr → decr
Rel Min: $(3,-64)$ $f(x)$ decr → incr

4. $f(x)=\frac{1}{2}x+\cos x \quad (0,2\pi)$

$f'(x)=\frac{1}{2}-\sin x$

$f'(x)=0$: $\frac{1}{2}-\sin x=0$, $\sin x=\frac{1}{2}$, $x=\frac{\pi}{6},\frac{5\pi}{6}$

~~f'(x) und~~ Not Poss

$f'(x)$: $+$ | $\pi/6$ | $-$ | $5\pi/6$ | $+$
$f(x)$: 0 $\nearrow$ $\pi/6$ $\searrow$ $5\pi/6$ $\nearrow$ 2π

Incr: $(0,\pi/6)\cup(5\pi/6,2\pi)$ $f'(x)>0$
Decr: $(\pi/6,5\pi/6)$ $f'(x)<0$
Rel Max: $(\pi/6,\ \pi/12+\sqrt{3}/2)$ $f(x)$ incr → decr
Rel Min: $(5\pi/6,\ 5\pi/12-\sqrt{3}/2)$ $f(x)$ decr → incr

5. $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32$

$\underline{f'(x) = 0}$

$4x^3 - 32 = 0$

$x^3 - 8 = 0$

$(x-2)\underbrace{(x^2+2x+4)}_{\text{No soln}} = 0$

$x = 2$

$\cancel{f'(x)}$ und

NOT Possible

$f'(x)$: $-$ | 2 | $+$

$f(x)$: ↘ 2 ↗

$$\boxed{\begin{array}{l} \text{Incr}: (2,\infty),\ f'(x)>0 \\ \text{Dec}: (-\infty,2),\ f'(x)<0 \\ \text{Rel Max: None} \\ \text{Rel Min}: (2,-44)\ f(x)\ \text{decr} \to \text{incr} \end{array}}$$

6. $f(x) = (x-1)^{\frac{1}{3}}$

$f'(x) = \dfrac{1}{3(x-1)^{2/3}}$

$\cancel{f'(x) = 0}$

NOT Poss.

$\underline{f'(x) \text{ und}}$

$3(x-1)^{2/3} = 0$

$x - 1 = 0$

$x = 1$

$f'(x)$: $+$ | 1 | $+$

$f(x)$: ↗ 1 ↗

$$\boxed{\begin{array}{l} \text{Incr}: (-\infty,1)\cup(1,\infty)\ f'(x)>0 \\ \text{Decr: None} \\ \text{Rel Max: None} \\ \text{Rel Min: None} \end{array}}$$

7. $f(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{2x(x-1) - x^2}{(x-1)^2}$

$f'(x) = \dfrac{2x^2 - 2x - x^2}{(x-1)^2}$

$f'(x) = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$

$\underline{f'(x) = 0}$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$\underline{f'(x) \text{ und}}$

$(x-1)^2 = 0$

$\cancel{x = 1}$ NOT in domain

$f'(x)$: $+$ | 0 | $-$ ⋮ 1 ⋮ $-$ | 2 | $+$

$f(x)$: ↗ 0 ↘ 1 ↘ 2 ↗

$$\boxed{\begin{array}{l} \text{Incr}: (-\infty,0)\cup(2,\infty),\ f'(x)>0 \\ \text{Decr}: (0,1)\cup(1,\infty),\ f'(x)<0 \\ \text{Rel Max}: (0,0)\ f(x)\ \text{incr} \to \text{decr} \\ \text{Rel Min}: (2,4)\ f(x)\ \text{decr} \to \text{incr} \end{array}}$$

Determine whether the statement is true or false. If false, explain why or give an example that shows it is false.

8. Every nth degree polynomial has $(n-1)$ critical numbers.

false $f(x) = (x-1)^4 \Rightarrow$ degree 4

$f'(x) = 4(x-1)^3$

Only 1 Critical #

9. An nth degree polynomial has at most $(n-1)$ critical numbers.

True!

10. There is a relative maximum or minimum at each critical #.

false $f(x) = x^3$

$f'(x) = 3x^2$

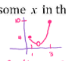
$x = 0$ crit # but NOT extrema

$f'(x)$: $+$ | 0 | $+$

$f(x)$: ↗ 0 ↗

11. The relative maxima of the function f are $f(1) = 4$ and $f(3) = 10$. f must have at least one minimum for some x in the interval $(1, 3)$.

false $\Rightarrow$



No min!

True if $f(x)$ is continuous!