Name_____ **Date**_____

Calc I H – 3.4 day 3 – Concavity & the 2nd Derivative Test **Period** _____

Find the points of inflection and discuss the concavity of the graph of the following functions.

1. $f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12$

$f''(x) = 12x - 6$

$f''(x) = 0$

$12x - 6 = 0$

$x = 1/2$

$f'(x)$ und — Not Poss

$f''(x)$: $-$ on $(-\infty, 1/2)$, $+$ on $(1/2, \infty)$

$f(x)$: $\cap$ on $(-\infty, 1/2)$, $\cup$ on $(1/2, \infty)$

Concave Up: $(1/2, \infty)$, $f''(x) > 0$

Concave down: $(-\infty, 1/2)$, $f''(x) < 0$

POI: $(1/2, -3/2)$, $f(x)$ changes Conc. or $f''(x)$ changes sign

2. $g(x) = x^3(x-4) = x^4 - 4x^3$

$g'(x) = 4x^3 - 12x^2$

$g''(x) = 12x^2 - 24x$

$g''(x) = 0$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0, 2$

$g''(x)$ und — Not Poss

$g''(x)$: $+$ on $(-\infty, 0)$, $-$ on $(0, 2)$, $+$ on $(2, \infty)$

$g(x)$: $\cup$ on $(-\infty, 0)$, $\cap$ on $(0, 2)$, $\cup$ on $(2, \infty)$

Concave Up: $(-\infty, 0) \cup (2, \infty)$, $g''(x) > 0$

Concave down: $(0, 2)$, $g''(x) < 0$

POI: $(0, 0)$, $(2, -16)$

$g(x)$ changes conc or $g''(x)$ changes sign

3. $h(x) = x + 2\cos x$; $(0, 2\pi)$

$h'(x) = 1 - 2\sin x$

$h''(x) = -2\cos x$

$h''(x) = 0$

$-2\cos x = 0$

$x = \pi/2, 3\pi/2$

$h''(x)$ und — Not Poss

$h''(x)$: $-$ on $(0, \pi/2)$, $+$ on $(\pi/2, 3\pi/2)$, $-$ on $(3\pi/2, 2\pi)$

$h(x)$: $\cap$ on $(0, \pi/2)$, $\cup$ on $(\pi/2, 3\pi/2)$, $\cap$ on $(3\pi/2, 2\pi)$

Concave Up: $(\pi/2, 3\pi/2)$, $h''(x) > 0$

Concave down: $(0, \pi/2) \cup (3\pi/2, 2\pi)$, $h''(x) < 0$

POI: $(\pi/2, \pi/2)$, $(3\pi/2, 3\pi/2)$

$h(x)$ Changes Concavity

$h''(x)$ Changes sign

Find all relative extrema. Use the Second Derivative Test where applicable.

4. $f(x) = x^3 - 9x^2 + 27x$

$f'(x) = 3x^2 - 18x + 27$

$f'(x) = 0$

$3x^2 - 18x + 27 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

Crit pt: $x = 3$

$f'(x)$ und — NOT Possible

$f''(x) = 6x - 18$

$f''(3) = 0 \Rightarrow$ Inconclusive

$f'(x)$: + | 3 | +

$f(x)$: ↗ 3 ↗

(3,27) NOT extrema!

No extrema

5. $g(x) = x + \frac{4}{x}$

$g'(x) = 1 - \frac{4}{x^2}$

$g'(x) = 0$

$x^2\left(1 - \frac{4}{x^2} = 0\right)$

$x^2 - 4 = 0$

Crit pts. $x = \pm 2$

$g'(x)$ und $x = 0$ VA — NOT extrema — Not in domain

$g''(x) = \frac{8}{x^3}$

$g''(2) = 1 > 0 \Rightarrow$ Conc. Up

(2,4) Rel Min Since $g'(2) = 0$ & $g''(2) > 0$

$g''(-2) = -1 < 0 \Rightarrow$ Conc. down

(-2,-4) Rel Max Since $g'(-2) = 0$ & $g''(-2) < 0$

6. $h(x) = 3x^5 - 20x^3$

$h'(x) = 15x^4 - 60x^2$

$h'(x) = 0$

$15x^4 - 60x^2 = 0$

$15x^2(x^2 - 4) = 0$

$x = 0, \pm 2 \Rightarrow$ crit pts.

$h'(x)$: + | −2 | − | 0 | − | 2 | +

$h(x)$: ↗ −2 ↘ 0 ↘ 2 ↗

$h'(x)$ und — NOT Poss

$h''(x) = 60x^3 - 120x$

$h''(0) = 0 \Rightarrow (0,0)$ NOT extrema Inconclusive

$h''(-2) = -240 < 0 \Rightarrow$ Conc down

(-2,64) Rel Max Since $h'(-2) = 0$ & $h''(-2) < 0$

$h''(2) = 240 > 0 \Rightarrow$ Conc Up

(2,-64) Rel Min Since $h'(2) = 0$ & $h''(2) > 0$

7. $p(x) = \cos x - x$; $(0, 4\pi)$

$p'(x) = -\sin x - 1$

$p'(x) = 0$

$-\sin x - 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}, \frac{7\pi}{2} \Rightarrow$ crit pts.

$p'(x)$ und — NOT poss

$p'(x)$: − | 0 | − | $\frac{3\pi}{2}$ | − | $\frac{7\pi}{2}$ | − | 4π

$p(x)$: 0 ↘ $\frac{3\pi}{2}$ ↘ $\frac{7\pi}{2}$ ↘ 4π

$p''(x) = -\cos x$

$p''\left(\frac{3\pi}{2}\right) = 0$

$p''\left(\frac{7\pi}{2}\right) = 0$

Inconclusive

NOT extrema

No extrema on interval!