

Name _____

Date _____

Calc I H - 3.4 day 3 - Concavity & the 2nd Derivative Test

Period _____

Find the points of inflection and discuss the concavity of the graph of the following functions.

1. $f(x) = 2x^3 - 3x^2 - 12x + 5$

$$f'(x) = 6x^2 - 6x - 12$$

$$f''(x) = 12x - 6$$

$$f''(x) = 0$$

$$12x - 6 = 0$$

$$x = \frac{1}{2}$$

Concave Up: $(\frac{1}{2}, \infty)$, $f''(x) > 0$ Concave down: $(-\infty, \frac{1}{2})$, $f''(x) < 0$ POI: $(\frac{1}{2}, -\frac{3}{2})$, $f(x)$ changes Conc.
or $f''(x)$ changes sign

2. $g(x) = x^3(x-4) = x^4 - 4x^3$

$$g'(x) = 4x^3 - 12x^2$$

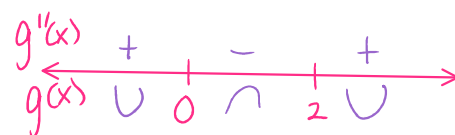
$$g''(x) = 12x^2 - 24x$$

$$g''(x) = 0$$

$$12x^2 - 24x = 0$$

$$12x(x-2) = 0$$

$$x = 0, 2$$

Concave Up: $(-\infty, 0) \cup (2, \infty)$, $g''(x) > 0$ Concave down: $(0, 2)$, $g''(x) < 0$ POI: $(0, 0)$, $(2, -16)$
 $g(x)$ changes Conc
or
 $g''(x)$ changes sign

3. $h(x) = x + 2\cos x$; $(0, 2\pi)$

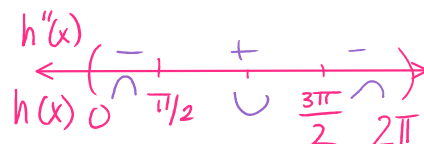
$$h'(x) = 1 - 2\sin x$$

$$h''(x) = -2\cos x$$

$$h''(x) = 0$$

$$-2\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

 ~~$h''(x)$ und~~
Not PossConcave Up: $(\frac{\pi}{2}, \frac{3\pi}{2})$, $h''(x) > 0$ Concave down: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$
 $h''(x) < 0$ POI: $(\frac{\pi}{2}, \frac{\pi}{2})$, $(\frac{3\pi}{2}, \frac{3\pi}{2})$
 $h(x)$ changes Concavity
 $h''(x)$ changes sign

Find all relative extrema. Use the Second Derivative Test where applicable.

4. $f(x) = x^3 - 9x^2 + 27x$

$f'(x) = 3x^2 - 18x + 27$

$f'(x) = 0$

$3x^2 - 18x + 27 = 0$

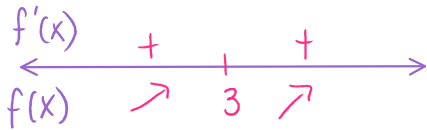
$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

Crit pt: $x=3$

$f''(x) = 6x - 18$

$f''(3) = 0 \Rightarrow$ Inconclusive



$(3, 27)$ NOT extrema!

No extrema

~~$f'(x)$ und~~
NOT Possible

5. $g(x) = x + \frac{4}{x}$

$g'(x) = 1 - \frac{4}{x^2}$

$g'(x) = 0$
 $x^2(1 - \frac{4}{x^2}) = 0$

$x^2 - 4 = 0$

Crit pts. $x = \pm 2$

$g''(x) = \frac{8}{x^3}$

$g''(2) = 1 > 0$
 \Rightarrow Conc. Up.

$g''(-2) = -1 < 0$
 \Rightarrow Conc. down

$(2, 4)$ Rel Min
Since $g'(2) = 0 \neq$
 $g''(2) > 0$

$(-2, -4)$ Rel Max
Since $g'(-2) = 0 \neq$
 $g''(-2) < 0$

~~$g'(x)$ und~~
 $x=0$ VA
NOT extrema -
Not in domain

6. $h(x) = 3x^5 - 20x^3$

$h'(x) = 15x^4 - 60x^2$

$h'(x) = 0$

$15x^4 - 60x^2 = 0$

$15x^2(x^2 - 4) = 0$

$x = 0, \pm 2 \Rightarrow$ Crit pts.

$h''(x) = 60x^3 - 120x$

$h''(0) = 0 \Rightarrow (0, 0)$ NOT extrema
Inconclusive

$h''(-2) = -240 < 0$
 \Rightarrow Conc down

$h''(2) = 240 > 0$
 \Rightarrow Conc Up

$(-2, 64)$ Rel Max
Since $h'(-2) = 0 \neq$
 $h''(-2) < 0$

$(2, -64)$ Rel Min
Since $h'(2) = 0 \neq$
 $h''(2) > 0$

~~$h'(x)$ und~~
NOT POSS



7. $p(x) = \cos x - x ; (0, 4\pi)$

$p'(x) = -\sin x - 1$

$p'(x) = 0$

$-\sin x - 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}, \frac{7\pi}{2} \Rightarrow$ Crit pts.

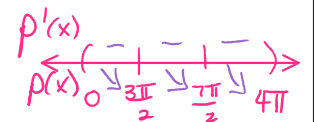
$p''(x) = -\cos x$

$p''(\frac{3\pi}{2}) = 0$

$p''(\frac{7\pi}{2}) = 0$

Inconclusive

~~$p'(x)$ und~~
NOT POSS



NOT extrema

No extrema
on interval!