Name_____ **Date**_____

Calc I H – 3.4 day 4 – Concavity & the 2nd Derivative Test **Period** _____

1. For the function $f(x)=\frac{16}{3}x^3+4x^2$ find any open intervals on which f is concave up and concave down. Find any points of inflection of f.

$f'(x)=16x^2+8x$

$f''(x)=32x+8$

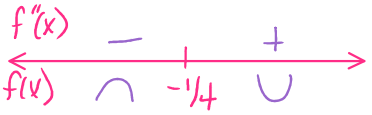
$f''(x)=0$

$32x+8=0$

$x=-1/4$

$f''(x)$ und

Not Poss.



Concave Up: $(-1/4,\infty)$, $f'(x)>0$

Concave down: $(-\infty,-1/4)$, $f'(x)<0$

$(-1/4, 1/6)$ POI ⇒ $f(x)$ Changes Conc.

2. Use the 2nd Derivative Test to find the relative extrema for $g(x)=8x^3-18x^2+2$

$g'(x)=24x^2-36x$

$g'(x)=0$

$24x^2-36x=0$

$12x(2x-3)=0$

$x=0, 3/2$ ⇒ Crit Pts

$g'(x)$ und

NP

$g''(x)=48x-36$

$g''(0)=-36<0$

Conc down

Rel max $(0,2)$ since $g'(0)=0$ & $g''(0)<0$

$g''(3/2)=36>0$

Conc up

Rel min $(3/2, -11\frac{1}{2})$ $g'(3/2)=0$ & $g''(3/2)>0$

For #3-6, find the points of inflection and discuss the concavity of the graph of the function.

3. $f(x)=x^{\frac{1}{3}}$

$f'(x)=\frac{1}{3}x^{-2/3}$

$f''(x)=-\frac{2}{9}x^{-5/3}$

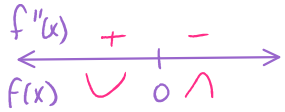
$=-\frac{2}{9x^{5/3}}$

$f''(x)=0$

ND

$f''(x)$ und

$x=0$



Conc up: $(-\infty,0)$, $f''(x)>0$

Conc down: $(0,\infty)$, $f''(x)<0$

POI $(0,0)$ ⇒ $f(x)$ changes concavity

4. $f(x)=2x^4-8x+3$

$f'(x)=8x^3-8$

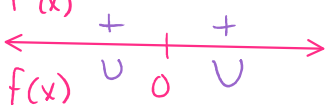
$f''(x)=24x^2$

$f''(x)=0$

$x=0$

$f''(x)$ und

NP



Conc up: $(-\infty,0)\cup(0,\infty)$, $f''>0$

Conc down: None

POI: None

5. $g(x)=2x^4-3x^2+2$

$g'(x)=8x^3-6x$

$g''(x)=24x^2-6$

$g''(x)=0$

$24x^2-6=0$

$4x^2-1=0$

$x=\pm\frac{1}{2}$

$g''(x)$ und

NP

Conc up: $(-\infty,-1/2)\cup(1/2,\infty)$ $g'(x)>0$

Conc down: $(-1/2, 1/2)$ $g''(x)<0$

POI $(-1/2, 1/8)$, $(1/2, 1/8)$

$g(x)$ changes conc

6. $y=x+\sin x$, $(0,2\pi)$

$y'=1+\cos x$

$y''=-\sin x$

$y''=0$

$x=\pi$

y'' und

NP

Conc up: $(\pi,2\pi)$, $y''>0$

Conc down: $(0,\pi)$, $y''<0$

POI: (π,π)

y changes conc.

For #7-10, find all intervals of increasing and decreasing. Identify relative extrema and justify using the 2nd Derivative Test. Discuss intervals of concavity and identify Points of Inflection.

7. $f(x)=x^2+3x-8$

$f'(x)=2x+3$

$f'(x)=0$ $\quad f'(x)$ und NP

$2x+3=0$

$x=-3/2$

$f''(x)=2$

$f''(x)>0$ for all x

Always Concave up!

$f'(x)$: $-$ on $(-\infty,-3/2)$, $+$ on $(-3/2,\infty)$; $f(x)$ ↘ $-3/2$ ↗

Incr: $(-3/2,\infty)$, $f'(x)>0$

Decr: $(-\infty,-3/2)$, $f'(x)<0$

Rel Min: $(-3/2,-10\frac{1}{4})$ since $f'(-3/2)=0$ & $f''(-3/2)>0$

Concave up: $(-\infty,\infty)$ $f''(x)>0$

Concave down: None

No POI!

8. $f(x)=-(x-5)^2$

$f'(x)=-2(x-5)$

$f'(x)=0$ $\quad f'(x)$ und NP

$-2(x-5)=0$

$x=5$

$f''(x)=-2$

$f''(x)<0$ for all x

Always Concave down!

$f'(x)$: $+$ on $(-\infty,5)$, $-$ on $(5,\infty)$; $f(x)$ ↗ 5 ↘

Incr: $(-\infty,5)$, $f'(x)>0$

Decr: $(5,\infty)$, $f'(x)<0$

Rel Max: $(5,0)$ since $f'(5)=0$ & $f''(5)<0$

Concave Up: None!

Concave down: $(-\infty,\infty)$, $f''(x)<0$

No POI!

9. $f(x)=x^3-2x^2+x-4$

$f'(x)=3x^2-4x+1$

$f'(x)=0$ $\quad f'(x)$ und NP

$3x^2-4x+1=0$

$(3x-1)(x-1)=0$

$x=1/3,\ 1$

$f'(x)$: $+$ on $(-\infty,1/3)$, $-$ on $(1/3,1)$, $+$ on $(1,\infty)$; $f(x)$ ↗ $1/3$ ↘ 1 ↗

Inc: $(-\infty,1/3)\cup(1,\infty)$ $f'(x)>0$

Dec: $(1/3,1)$ $f'(x)<0$

Rel Max: $(1/3,-10\frac{4}{27})$ $f'(1/3)=0$, $f''(\frac{1}{3})=2-4<0$

Rel Min: $(1,-4)$ $f'(1)=0$, $f''(1)=2>0$

$f''(x)=6x-4$

$f''(x)=0$ $\quad f''(x)$ und NP

$6x-4=0$

$x=\frac{2}{3}$

$f''(x)$: $-$ on $(-\infty,2/3)$, $+$ on $(2/3,\infty)$; $f(x)$ ∩ $2/3$ ∪

Concave Up: $(2/3,\infty)$ $f''(x)>0$

Conc down: $(-\infty,2/3)$ $f''(x)<0$

POI: $(2/3,-10\frac{4}{27})$ $f(x)$ changes Concavity

10. $f(x)=\sqrt{x^2+1}$

$f'(x)=\frac{1}{2\sqrt{x^2+1}}\cdot 2x=\frac{x}{\sqrt{x^2+1}}$

$f'(x)=0$ $\quad f''(x)$ und $x^2+1=0$ NP

$x=0$

$f'(x)$: $-$ on $(-\infty,0)$, $+$ on $(0,\infty)$; $f(x)$ ↘ 0 ↗

Incr: $(0,\infty)$, $f'(x)>0$

Decr: $(-\infty,0)$, $f'(x)<0$

Rel Min: $(0,1)$ $f'(0)=0$, $f''(0)=1>0$

$f''(x)=\dfrac{(x^2+1)^{1/2}-\frac{1}{2}(x^2+1)^{-1/2}(2x)(x)}{(x^2+1)}$

$f''(x)=\dfrac{(x^2+1)^{-1/2}(x^2+1-x^2)}{(x^2+1)}$

$f''(x)=\dfrac{1}{(x^2+1)^{3/2}}$

$f''(x)=0$ NP $\quad f''(x)$ und NP

$f''(x)$: $+$ on $(-\infty,\infty)$; $f(x)$ ∪

Conc up: $(-\infty,\infty)$

Conc down: None

No POI

No POI!