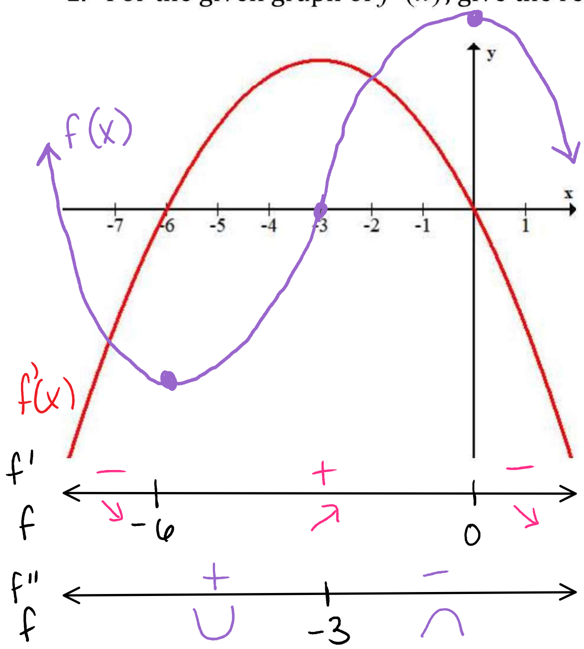
Name Answer Key                                        Date

**Calc I H – 3.4 Graphing Review**                                        **Period**

1. For the given graph of $f'(x)$, give the requested information **AND** sketch the graph of $f(x)$:




(a) Interval(s) on which $f(x)$ is increasing: $(-6, 0)$

(b) Interval(s) on which $f(x)$ is decreasing: $(-\infty, -6) \cup (0, \infty)$

(c) Relative maxima of $f(x)$ at $x =$ $0$

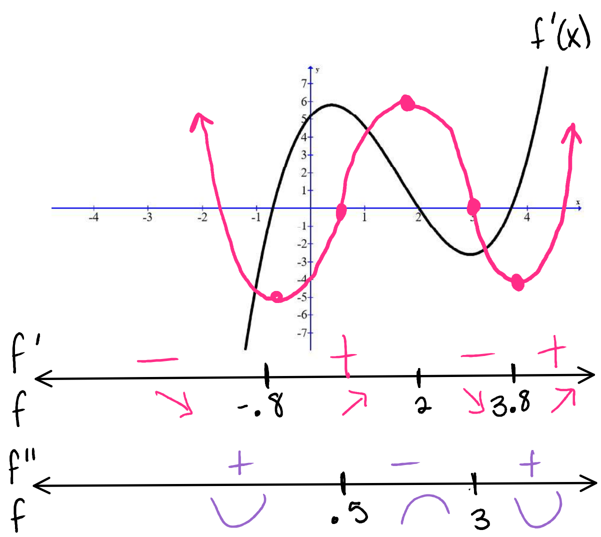
(d) Relative minima of $f(x)$ at $x =$ $-6$

(e) Interval(s) on which $f(x)$ is concave up: $(-\infty, -3)$

(f) Interval(s) on which $f(x)$ is concave down: $(-3, \infty)$

(g) Point(s) of inflection of $f(x)$ at $x =$ $-3$

2. For the given graph of $f'(x)$, give the requested information **AND** sketch the graph of $f(x)$:




(a) Interval(s) on which $f(x)$ is increasing: $(-.8, 2) \cup (3.8, \infty)$

(b) Interval(s) on which $f(x)$ is decreasing: $(-\infty, -.8) \cup (2, 3.8)$

(c) Relative maxima of $f(x)$ at $x =$ $2$

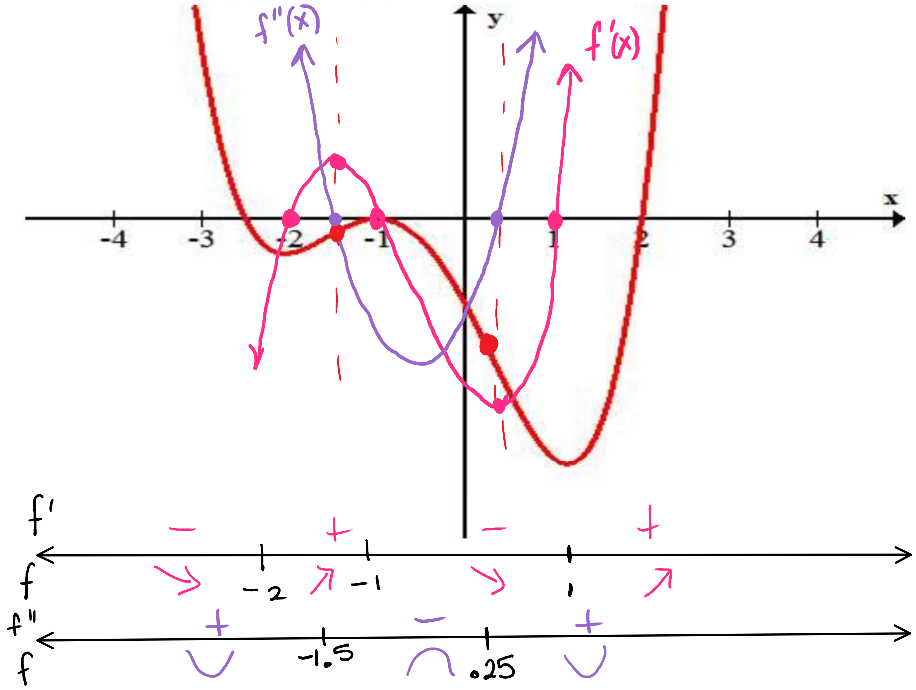
(d) Relative minima of $f(x)$ at $x =$ $-.8,\ 3.8$

(e) Interval(s) on which $f(x)$ is concave up: $(-\infty, .5) \cup (3, \infty)$

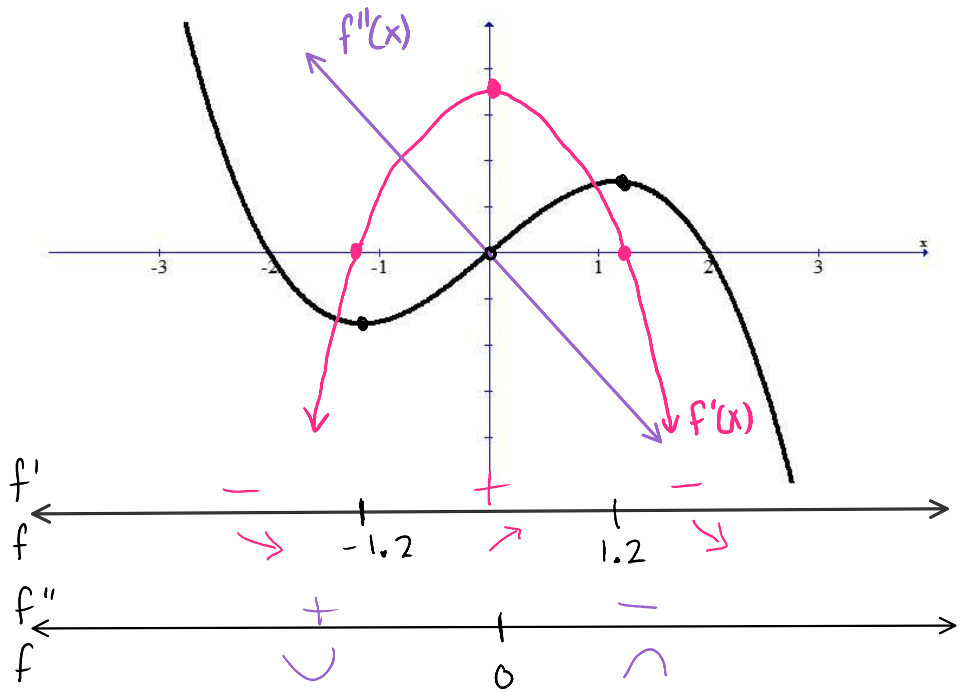
(f) Interval(s) on which $f(x)$ is concave down: $(.5, 3)$

(g) Point(s) of inflection of $f(x)$ at $x =$ $.5,\ 3$

3. Given the graph of $f(x)$, <u>sketch</u> the graphs of $f'(x)$ and $f''(x)$ on the same set of axes.
**<u>Plot</u> the important points on all 3 graphs to show which points line up at the same *x*-coordinate.**
Clearly <u>label</u> each graph as $f'(x)$ and $f''(x)$. *Showing work is recommended but optional.*




4. Given the graph of $f(x)$, <u>sketch</u> the graphs of $f'(x)$ and $f''(x)$ on the same set of axes.
**<u>Plot</u> the important points on all 3 graphs to show which points line up at the same *x*-coordinate.**
Clearly <u>label</u> each graph as $f'(x)$ and $f''(x)$. *Showing work is recommended but optional.*



5. In the following cases, how is the graph of $f(x)$ affected?

a) The graph of $f'(x)$ is above the $x$-axis f(x) increases

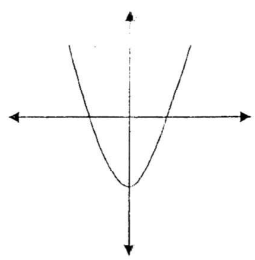
b) The graph of $f'(x)$ crosses the $x$-axis at $x = c$ extrema on f(x)

c) The graph of $f'(x)$ is below the $x$-axis f(x) decreases

6. At a relative maximum or minimum of the graph of $f(x)$, what can you conclude about the graph of $f'(x)$? f'(x) has zeroes or x-intercepts.

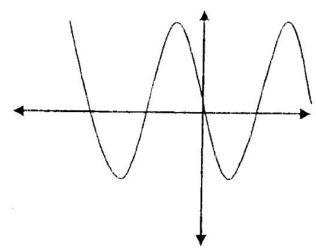
**Match the graphs of the function in #7-10 with the graph of their derivatives (a) – (d) below.**

7.



c

(a)



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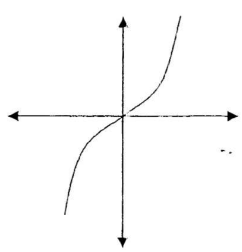
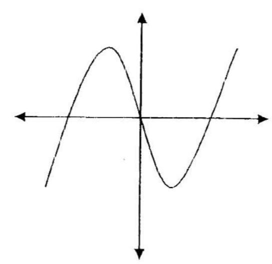


d

(b)



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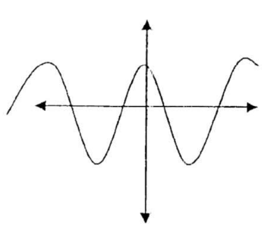
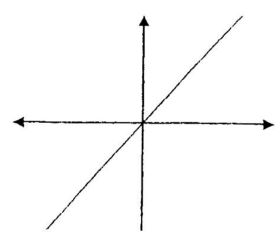
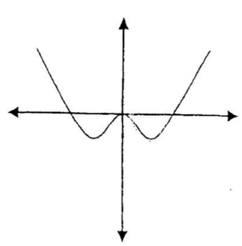


a

(c)

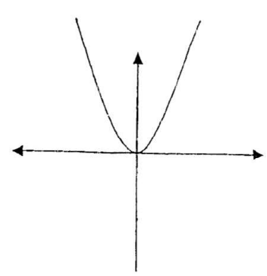


10.



b

(d)



11. Sketch a possible $f(x)$ given the following information:

a) $f$ is continuous on $(-\infty,\infty)$

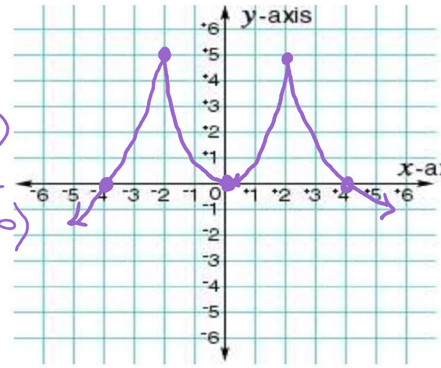
$f(-4)=0 \quad f(0)=0 \quad f(4)=0$ $\quad$ (-4,0) (0,0) (4,0)

$f'(x)>0$ if $x<-2$ and $0<x<2$ $\quad$ incr $(-\infty,-2)\cup(0,2)$

$f'(-2)$ and $f'(2)$ does not exist $\quad$ Non diff. at $x=\pm 2$

$f'(x)<0$ if $-2<x<0$ *and* $x>2$ $\quad$ decr $(-2,0)\cup(2,\infty)$

$f''(x)>0$ if $x\neq\pm 2$ $\quad$ Concave up except at $x\neq\pm 2$

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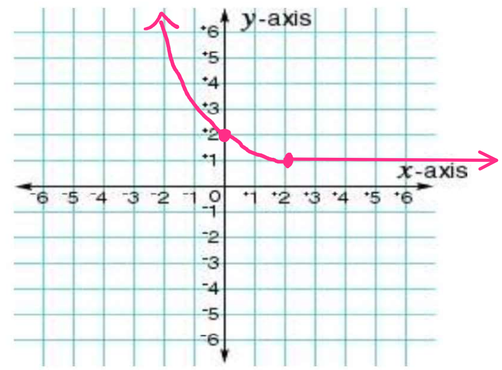
b) $f$ is continuous on $(-\infty,\infty)$

$f'(x)<0$ when $x<2$ $\quad$ decr $(-\infty,2)$

$f''(x)>0$ when $x<2$ $\quad$ conc up $(-\infty,2)$

$f(x)=1$ when $x\geq 2$ $\quad$ $y=1$ $(2,\infty)$

$f(0)=2$ $\quad$ (0,2)

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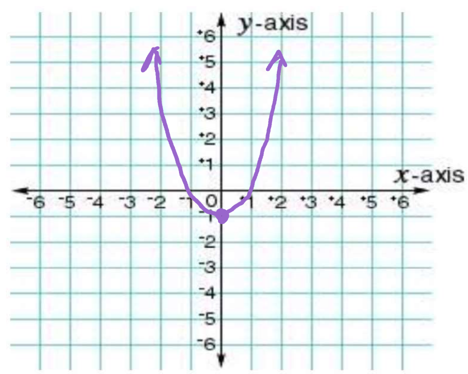
c) $f$ is continuous on $(-\infty,\infty)$

$f'(x)<0$ when $x<0$ $\quad$ decr $(-\infty,0)$

$f'(x)>0$ when $x>0$ $\quad$ incr $(0,\infty)$

$f''(x)>0$ $\quad$ Concave up

$f(0)=-1$ $\quad$ (0,-1)

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d) $f$ is continuous on $(-\infty,\infty)$

$f'(x)>0$ when $x<0$ or $x>3$ $\quad$ incr $(-\infty,0)\cup(3,\infty)$

$f'(x)<0$ when $0<x<3$ $\quad$ decr $(0,3)$

$f'(0)=0$ $\quad$ max at $x=0$ (incr $\rightarrow$ decr)

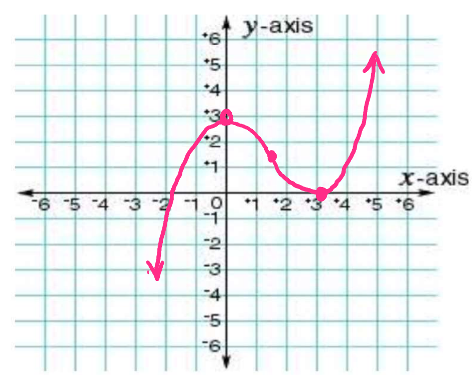
$f(0)=3$ $\quad$ (0,3)

$f(3)=0$ $\quad$ (3,0)

$f''(x)<0$ when $x<1.5$ $\quad$ Concave down $(-\infty,1.5)$

$f''(x)>0$ when $x>1.5$ $\quad$ Concave up $(1.5,\infty)$

$\therefore$ POI at $x=1.5$



12. Analyze and sketch the graph of $f(x)=(x^3-8)^{\frac{2}{3}}$.

Given: $f'(x)=\dfrac{2x^2}{(x^3-8)^{\frac{1}{3}}}$

$f'(x)=0$

$2x^2=0$

$x=0$

$f'(x)$ und

$x^3-8=0$

$x=2$

$f''(x)=\dfrac{2x^4-32x}{(x^3-8)^{\frac{4}{3}}}$

$f''(x)=0$

$2x^4-32x=0$

$2x(x^3-16)=0$

$x=0 \quad x^3-16=0$

$x^3=16$

$x=\sqrt[3]{16}$

$x\approx 2.520$

$f''(x)$ und

$x^3-8=0$

$x=2$

$f(x)=(x^3-8)^{2/3}=0$

$x=2$

$f(0)=(-8)^{2/3}=4$

Domain: $(-\infty,\infty)$

*x*-intercept(s): $(2,0)$

*y*-intercept: $(0,4)$

Vertical asymptote(s): none

Horizontal asymptote(s): none

Slant asymptote(s): none

Incr: $(2,\infty)$ $f'>0$

Decr: $(-\infty,0)\cup(0,2)$ $f'<0$

Extrema: $(2,0)$ Rel min

$f'<0 \rightarrow f'>0$

Conc Up: $(-\infty,0)\cup(2.520,\infty)$ $f''>0$

Conc Down: $(0,2)\cup(2,2.520)$ $f''<0$

POI: $(0,4)$, $(2.520,4)$

$f''$ changes sign