1. Find all antiderivatives of the function $\int f(x) = \int 3x^2 + \sec^2 x - 4e^x \, dx$

   $F(x) = x^3 + \tan x - 4e^x + C$

   a. $\boxed{x^3 + \tan x - 4e^x + C}$
   b. $x^3 + \tan x - 2e^{2x} + C$
   c. $x^3 + \dfrac{\sec^3 x}{3} - 2e^{2x} + C$
   d. $x^3 + 2\sec^2 x \tan x - 4e^x + C$

2. A particle moves along the *x*-axis with velocity $v(t) = t^2 + t, t \geq 0$. If the particle is at $x = -1$, when $t = 0$, then what is the position x of the particle at time $t = 3$?

   $x(t) = \int (t^2 + t)\,dt = \frac{t^3}{3} + \frac{t^2}{2} + C$
   $x(0) = C = -1$
   $x(t) = \frac{t^3}{3} + \frac{t^2}{2} - 1$
   $x(3) = \frac{27}{3} - \frac{9}{2} - 1 = 9 + \frac{9}{2} - 1 = \frac{25}{2}$

   a. -1 b. $\dfrac{29}{2}$ c. $\boxed{\dfrac{25}{2}}$ d. 12

3. The acceleration a of an object as it moves along the *x*-axis is given by $a(t) = 2 + 12t$, and the velocity v of the object at time $t = 0$ is 5. If $s(t)$ is the distance of the object from the origin at time t, find $s(3) - s(1)$.

   $v(t) = \int (2 + 12t)\,dt = 2t + 6t^2 + C$
   $v(0) = C = 5$

   a. 86 b. $\boxed{70}$ c. 30 d. 52

   $v(t) = 2t + 6t^2 + 5$

   $\int_1^3 v(t)\,dt = s(3) - s(1) \Rightarrow \int_1^3 (2t + 6t^2 + 5)\,dt = \left[t^2 + 2t^3 + 5t \right]_1^3$
   $= 9 + 2(27) + 5(3) - (1 + 2 + 5) = 9 + 54 + 15 - 1 - 2 - 5 = 70$

4. $\displaystyle\int_1^e \frac{3x^2 + 1}{x}\,dx = \int_1^e \left(3x + \frac{1}{x}\right) dx = \left(\frac{3x^2}{2} + \ln x\right)_1^e = \frac{3e^2}{2} + \ln e - \left(\frac{3}{2} + \cancel{\ln 1}\right)$
   $= \frac{3e^2}{2} + 1 - \frac{3}{2} = \frac{3e^2 - 1}{2}$

   a. $\boxed{\dfrac{3e^2 - 1}{2}}$ b. $\dfrac{3e^2 + 1}{2}$ c. $e^3 + 1$ d. $3e^2 - 1$

5. $\displaystyle\int_1^4 \sqrt{x}\left(x - \frac{1}{x}\right) dx = \int_1^4 \left(x^{3/2} - x^{-1/2}\right) dx = \left[\frac{2}{5} x^{5/2} - 2x^{1/2}\right]_1^4$

   a. $\dfrac{32}{5}$ b. $\dfrac{44}{5}$ c. $\boxed{\dfrac{52}{5}}$ d. $\dfrac{56}{15}$

   $= \frac{2}{5}(4)^{5/2} - 2(4)^{1/2} - \frac{2}{5} + 2 = \frac{2}{5}(32) - 4 - \frac{2}{5} + 2 = \frac{64}{5} - 4 - \frac{2}{5} + 2$
   $= \frac{62}{5} - 2 = \frac{62}{5} - \frac{10}{5} = \frac{52}{5}$

$\pi/2$

6. The average value of $f(x) = \sin x$ on the interval $\left[-\frac{\pi}{3}, \frac{\pi}{2}\right]$ is

$Avf = \frac{1}{\frac{3\pi}{2(3)} - \left(\frac{-\pi(2)}{3(2)}\right)} \int_{-\pi/3}^{\pi/2} \sin x \, dx$

$= \frac{1}{5\pi/6} (-\cos x)\Big|_{-\pi/3}^{\pi/2} = \frac{6}{5\pi}\left[-\cos\frac{\pi}{2} + \cos\left(\frac{-\pi}{3}\right)\right]$

a. $-\frac{3}{5\pi}$ b. $\boxed{\frac{3}{5\pi}}$ c. $\frac{1}{2}$ d. $\frac{5\pi}{12}$

$= \frac{6}{5\pi}\left[0 + \frac{1}{2}\right)$

$= \frac{3}{5\pi}$

7. An object moves along the x-axis with velocity v meters per second. If $v(t) = 3t^2 + t - 2$, what is the average velocity of the object during the interval $0 \le t \le 6$?

$\frac{1}{6}\int_0^6 (3t^2 + t - 2)\,dt$

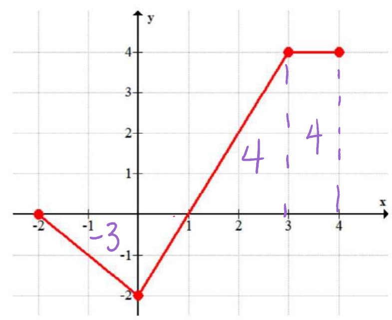
a. $\boxed{\text{37 m/s}}$ b. 38 m/s c. 41 m/s d. 222 m/s

$= \frac{1}{6}\left(t^3 + \frac{t^2}{2} - 2t\right)\Big|_0^6$

$= \frac{1}{6}\left(216 + \frac{36}{2} - 12\right)$

$= \frac{222}{6} = 37$

8. The graph of the piecewise function f is below. What is $\int_{-2}^{4} f(x)\,dx$?



$\int_{-2}^{4} f(x)\,dx = -3 + 4 + 4 = 5$

a. 2 b. $\boxed{5}$ c. $\frac{17}{2}$ d. $\frac{56}{15}$

9. If $f(x) = \begin{cases} x^3, & x \le 1 \\ \frac{1}{x}, & x > 1 \end{cases}$ then $\int_0^e f(x)\,dx =$

$\int_0^1 x^3\,dx + \int_1^e \ln x\,dx = \frac{x^4}{4}\Big|_0^1 + \ln x\Big|_1^e$

$= \frac{1}{4} - 0 + \ln e - \ln 1$

a. 0 b. $\boxed{\frac{5}{4}}$ c. $\frac{1}{4} + e$ d. $\frac{e^4}{4}$

$= \frac{1}{4} + 1 = \frac{5}{4}$

10. Find $\int_{-2}^{6} f(x)\,dx$ when $f(x) = \begin{cases} -x, & -2 \le x < 2 \\ x+3, & 2 \le x \le 6 \end{cases}$

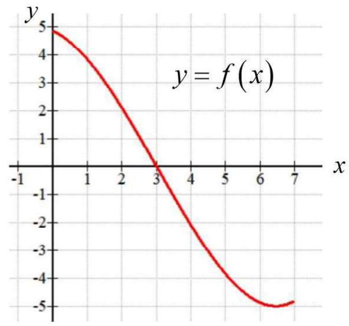
$= \int_{-2}^{2} -x\,dx + \int_2^6 (x+3)\,dx$

a. 24 b. $\boxed{28}$ c. 32 d. 44

$= -\frac{x^2}{2}\Big|_{-2}^{2} + \left(\frac{x^2}{2} + 3x\right)\Big|_2^6$

$= \cancel{-\frac{4}{2} - \left(\frac{-4}{2}\right)}^{0} + \frac{36}{2} + 18 - \frac{4}{2} - 6 = 18 + 18 - 8 = 28$

11. The graph of a function f that is differentiable is shown below. If $F(x)=\int_0^x f(t)\,dt$, which of the following is true?



$F' = f(x)$

$F'' = f'(x)$

$F(3) > 0$

$F'(3) = f(3) = 0$

$F''(3) = f'(3) < 0$

a. $F(3) < F'(3) < F''(3)$

b. $F'(3) < F''(3) < F(3)$

c. $F''(3) < F(3) < F'(3)$

d. $\boxed{F''(3) < F'(3) < F(3)}$

12. An object is moving along the x-axis. If its velocity v at time t (in minutes) is $v(t)=t^2-2t$ (in ft/min), what is the total distance the object travels between $t=0$ and $t=4$ minutes?

a. $0\ ft$

b. $\boxed{\frac{16}{3}\ ft}$

c. $8\ ft$

d. $\frac{40}{3}\ ft$

$$\int_0^4 (t^2-2t)\,dt = \left[\frac{t^3}{3}-t^2\right]_0^4 = \frac{64}{3}-16-0 = \frac{64}{3}-\frac{48}{3} = \frac{16}{3}$$

13. If at every point (x, y) on the graph of a function f, the slope of the tangent line is given by $y = 3-4x$ and if the point $(2, 3)$ is on the graph of f, then

$\int \frac{dy}{dx}dx = \int 3-4x\,dx$

$y = 3x - 2x^2 + C$

$3 = 6 - 8 + C$

$3 = -2 + C$

$C = 5$

a. $f(x) = -5x+7$

b. $f(x) = -2x^2+3x$

c. $f(x) = -2x^2+3x-11$

d. $\boxed{f(x) = -2x^2+3x+5}$

14. $\int \frac{x^2-3x+2\sqrt{x}-1}{x}\,dx =$ $\int (x - 3 + 2x^{-1/2} - x^{-1})\,dx = \frac{x^2}{2} - 3x + 4\sqrt{x} - \ln x + C$

a. $\boxed{\frac{1}{2}x^2 - 3x + 4x^{1/2} - \ln|x| + C}$

b. $\frac{1}{2}x^2 - 3x + \frac{6}{5}x^{5/2} - \ln|x| + C$

c. $\frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{4}{3}x^{3/2} - \ln|x| + C$

d. $\frac{1}{2}x^2 - 3x + x^{1/2} - \ln|x| + C$

15. Given $\frac{d^2y}{dx^2} = 3x^2 - 6x$:

a. Find the general solution to the differential equation.

$\frac{dy}{dx} = x^3 - 3x^2 + C_1$

$y = \frac{x^4}{4} - x^3 + C_1 x + C_2$

b. Verify the solution found in part (a).

$\frac{d}{dx}\left(y = \frac{x^4}{4} - x^3 + C_1 x + C_2\right) \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx} = x^3 - 3x^2 + C_1\right) \Rightarrow \frac{d^2y}{dx^2} = 3x^2 - 6x$ ✓

c. Find the particular solution to the differential equation given the conditions when $x = 0$, then $y = 2$ and when $x = 1$, then $y = 3$.

$(0,2)$

$2 = C_2$

$y = \frac{x^4}{4} - x^3 + C_1 x + 2$

$(1,3)$

$3 = \frac{1}{4} - 1 + C_1 + 2$

$3 = \frac{5}{4} + C_1$

$C_1 = \frac{7}{4}$

$\boxed{y = \frac{x^4}{4} - x^3 + \frac{7}{4}x + 2}$

16. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure below. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

a. Find $g(-4)$. Find $g'(x)$ and evaluate $g'(-4)$.

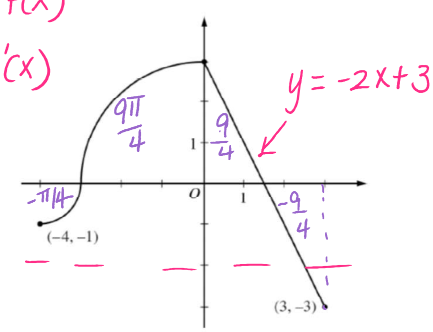
$g'(x) = 2 + f(x)$

$g''(x) = f'(x)$

$g(-4) = 2(-4) + \int_0^{-4} f(t)\,dt = -8 - \int_{-4}^{0} f(t)\,dt$

$= -8 - \left(-\pi/4 + 9\pi/4\right) = -8 - 2\pi$

$g'(-4) = 2 + f(-4) = 2 + (-1) = 1$



Graph of f

b. Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

$g''(x) = 0$ at $x = 0$

$g''(x)$ ~~und at~~ $x = -3$
NOT POI

g'': + + −
g: ∪ ∪ ∩
−4 −3 0 3

g has POI at $x = 0$ since g' goes from incr → decr or $g'' > 0 \Rightarrow g'' < 0$.

c. Determine the x-coordinate of the point at which g has a critical point(s) on the interval $-4 \le x \le 3$. Classify the critical point as an absolute maximum, absolute minimum or neither. Justify your answer.

$g'(x) = 2 + f(x) = 0$

$f(x) = -2$

$-2x + 3 = -2$

$-2x = -5$

$x = \frac{5}{2}$

Abs max at $x = \frac{5}{2}$ since $g'(x)$ entirely positive on $(-4, 5/2)$ and entirely negative on $(5/2, 3)$.