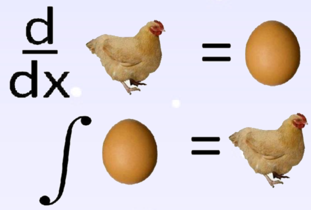
4.1 day 1 - Antiderivatives and Indefinite Integration 1/30/19



Homework:

- Section 4.1 A

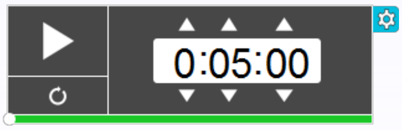
Objective:

Write the general solution of a differential equation; Use indefinite integral notation for antiderivatives; Use basic integration rules to find antiderivatives.

Do Now:

Discuss the *Do Now* in your groups.

How many different answers could there be for each differential equation?



AP CALCULUS AB **Section 4.1:** Antiderivatives and Indefinite Integration **Day 1**

Do Now:

1. Find the function $f(x)$ given $\frac{d}{dx}f(x)$ below.

a. $\frac{d}{dx}f(x)=3x^2$ b. $\frac{d}{dx}\overset{g}{\cancel{f}}(x)=\cos(x)$ c. $\frac{d}{dx}\overset{h}{\cancel{f}}(x)=\frac{1}{x}$ d. $\frac{d}{dx}\overset{j}{\cancel{f}}(x)=2xe^{x^2}$

2. Are these the only possible answers? Justify your answers.

$f(x)=x^3$ $g(x)=\sin x$ $h(x)=\ln x$ $j(x)=e^{x^2}$

$f(x)=x^3+4$ $g(x)=\sin x+3$ $h(x)=\ln x+\pi$ $j(x)=e^{x^2}+5$

$f(x)=x^3+8$ $g(x)=\sin x-\pi$ $h(x)=\ln(2x)$ $=\underbrace{\ln 2}_{C}+\ln x$ $j(x)=e^{x^2}+e$

$f(x)=x^3+C$

Vert shift

For each example above, write a different function whose derivative is given.

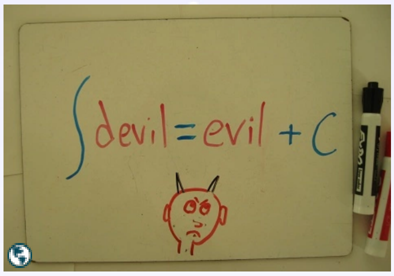
Definition of an Antiderivative

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example 1: Given $f(x) = -\sin(x)$, what function $F(x)$ represents the family of antiderivatives of $f(x)$?

Constant of Integration

$$F(x) = \cos x + C$$



The family of functions represented by F is called the ***General Antiderivative.***

What is the relationship between integrating and differentiating?

A **differential equation** is an equation that involves variables and their derivatives.

Example 2: Find the general solution of the differential equation $y' = sec^2(x)$.

What are we solving for?

$$y = \tan x + C$$

Notation

The family of all antiderivatives of the function $f(x)$ is indicated by $\int f(x)\,dx = F(x)+C$.

Variable of Integration

Constant of Integration

$$y = \int \underline{f(x)}\,dx = F(x)+C$$

Integrand

antiderivative

$\sqrt{f(x)}$

- The expression $\int f(x)$ is read as the **antiderivative of f with respect to x.**
- The differential dx serves to identify x as the variable of integration.
- The term **indefinite integral** is a synonym for antiderivative.

You use the inverse nature of integration and differentiation to solve.

$$y' = \sec^2 x$$

$$\frac{dy}{dx} = \sec^2 x$$

$$dy = \sec^2 x\,dx$$

$$\int \underline{dy} = \int \sec^2 x\,dx$$

$$y = \int \sec^2 x\,dx$$

$$y = \tan x + C$$

See table on p. 250!

$$\int 1\,dy = y + c$$

$$\int d\,evil = evil + c$$

Examples:

Evaluate each indefinite integral.

1. $\int 7\,dx$

$= 7x + C$

2. $\int x^5\,dx$

$= \dfrac{x^6}{6} + C$

3. $\int x^{\frac{2}{3}}\,dx$

$= \dfrac{x^{5/3}}{5/3} + C$

$= \dfrac{3}{5}x^{5/3} + C$

How can we check our answers?

What does the "+ c" at the end represent graphically?

4. $\int (x^4 - x^2)dx$

5. $\int (t^4 - t^3 + t)dt$

$= \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^2}{2} + C$

6. $\int \frac{1}{1+x^2}dx$

$= \arctan x + C$

Where have we seen the pattern in #6 before?

You MUST know your formulas for differentiating!!

7. $\int (e^x - 7x - 8)dx$

$= e^x - \frac{7}{2}x^2 - 8x + C$

8. $\int \left(\frac{3}{4}a^5 + \frac{5}{3}a^2 - \frac{a}{2}\right)da$

9. $\int \left(\pi x + \frac{1}{\pi}\right)dx$

$= \frac{\pi}{2}x^2 + \frac{1}{\pi}x + C$

10. $\int \frac{1}{x^2}dx = \int x^{-2}dx$

$= -x^{-1} + C$

$= -\frac{1}{x} + C$

11. $\int \frac{1}{x}dx = \cancel{\int x^{-1}} + C$

$= \ln x + C$

$\cancel{\frac{x^0}{0}}$

12. $\int 4x^5dx$

Process:
Original ⟶ *Rewrite* ⟶ *Integrate* ⟶ *Simplify*

13. $\int \sqrt{n}\,dn = \int x^{1/2}dx$

$= \frac{2}{3}x^{3/2} + C$

14. $\int \frac{4}{x^3} - \frac{1}{x^4}dx$

15. $\int 4x^{\pi}dx$

$= \frac{4x^{\pi+1}}{\pi+1} + C$

16. $\int (2x-3)^2\,dx$

$\int (4x^2-12x+9)\,dx$

$= \frac{4}{3}x^3 - 6x^2 + 9x + C$

17. $\int \frac{x^2+3x+1}{x^4}\,dx$

$\int (x^{-2}+3x^{-3}+x^{-4})\,dx$

$= -x^{-1} + \frac{3}{-2}x^{-2} + \frac{x^{-3}}{-3} + C$

$= -\frac{1}{x} - \frac{3}{2x^2} - \frac{1}{3x^3} + C$

18. $\int \frac{(2x-5)(3x+2)}{\sqrt{x}}\,dx$

$\int \frac{6x^2-11x-10}{x^{1/2}}\,dx$

$\int (6x^{3/2} - 11x^{1/2} - 10x^{-1/2})\,dx$

$= \frac{12}{5}x^{5/2} - \frac{22}{3}x^{3/2} - 20x^{1/2} + C$

19. $\int 4\sin(x)\,dx$

$= -4\cos x + C$

20. $\int \frac{-2\cos x}{3}\,dx$

21. $\int \frac{5}{\cos^2(x)}\,dx$

$\int 5\sec^2 x\,dx$

$= 5\tan x + C$

22. $\int (4\cos(x) - 9\sin(x))dx$

23. $\int \frac{-\sin x}{\cos^2 x} dx$

24. $\int \theta^2 - 2\csc^2\theta \, d\theta$

$$\frac{-\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\int -\tan x \sec x \, dx$$

$$= -\sec x + C$$

25. Find the particular solution to $f'(x) = (2x-3)^2$ if $f(1) = 10$.

$$\int (2x-3)^2 dx$$

$$= \frac{4}{3}x^3 - 6x^2 + 9x + C$$

$$f(x) = \frac{4}{3}x^3 - 6x^2 + 9x + C$$

$$f(1) = \frac{4}{3} - 6 + 9 + C = 10$$

$$4 - 18 + 27 + 3C = 30$$

$$-5 + 3C = 30$$

$$3C = 35$$

$$C = \frac{35}{3}$$

$$\boxed{f(x) = \frac{4}{3}x^2 - 6x^2 + 9x + \frac{35}{3}}$$