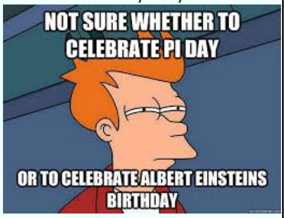
4.1 day 1 - Antiderivatives and Indefinite Integration 3/14/19



Homework:

- 4.1A
- 4.1 Quiz - Thursday, 3/21 or Friday, 3/22

Objective: Write the general solution of a differential equation; Use indefinite integral notation for antiderivatives; Use basic integration rules to find antiderivatives.

Do Now: Given the following derivatives, find the original functions:

1) $f'(x) = 2x$ 2) $f'(x) = 3x^2$ 3) $f'(x) = x$

$f(x) = x^2 + 6 - 5$

$f(x) = x^2$

$f(x) = x^2 + C$

$f(x) = x^2 + 1$

$f(x) = x^2 - 1{,}000{,}000$

$f(x) = x^3$

$f(x) = x^3 + \pi$

$f(x) = x^3 + C$

$f(x) = \frac{x^2}{2}$

BUT... before we begin... for your entertainment... I bring you...

a musical π day song!

Definition of an Antiderivative

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Integrating is the process of finding the antiderivative.

How else might we describe the relationship between integrating and differentiating?

Notation

The family of all antiderivatives of the function $f(x)$ is indicated by $\int f(x)\,dx = F(x) + C$.

Variable of Integration

Constant of Integration

$$y = \int \underbrace{f(x)}_{\text{Integrand}} dx = F(x) + C$$

The expression $\int f(x)$ is read as the *antiderivative of f with respect to x.*

The differential dx serves to identify x as the variable of integration.

The term **indefinite integral** is a synonym for antiderivative.

Let's write our first rule for integration on the board.

$$\frac{d}{dx} x^n = n x^{n-1}$$

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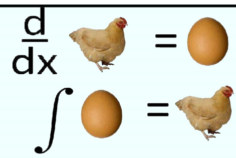
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Let's Practice

Find the indefinite integral



$7 = 7x^0$

1) $\int (x - 7)\,dx$

$$= \frac{x^2}{2} - 7x + C$$

2) $\int (5x^4 - 8x^3 + 3)\,dx$

$$= \frac{5x^5}{5} - \frac{8x^4}{4} + 3x + C$$

$$= x^5 - 2x^4 + 3x + C$$

Let's Practice

Find the indefinite integral

3) $\int (\sqrt[3]{x^4} + \sqrt{x})\,dx$

$$\int \left(x^{\frac{4}{3}} + x^{\frac{1}{2}}\right) dx$$

$$= \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{3}{7}x^{\frac{7}{3}} + \frac{2}{3}x^{\frac{3}{2}} + C$$

4) $\int (\sqrt[5]{x} + 2\sqrt[3]{x^2})\,dx$

$$\int \left(x^{\frac{1}{5}} + 2x^{\frac{2}{3}}\right) dx$$

$$= \frac{5}{6}x^{\frac{6}{5}} + 2 \cdot \frac{3}{5}x^{\frac{5}{3}} + C$$

$$= \frac{5}{6}x^{\frac{6}{5}} + \frac{6}{5}x^{\frac{5}{3}} + C$$

Let's Practice

Find the indefinite integral

$(3w^2+4)(3w^2+4)$

5) $\int \frac{5x^3+2x^2-4}{x^2}dx$

6) $\int (3w^2+4)^2\,dw$

$\int (5x+2-4x^{-2})dx$

$\frac{-4x^{-1}}{-1}$

$= \int (5x+2-4x^{-2})dx$

$= \frac{5}{2}x^2+2x+\frac{4}{x}+c$

$\int (9w^4+24w^2+16)dw$

$= \int (9w^4+24w^2+16)dw$

$= \frac{9}{5}w^5+8w^3+16w+c$

Trig Integration

$\int \sin(x)dx = -\cos x + C$

$\int \sec^2(x)dx = \tan x + C$

$\int \cos(x)dx = \sin x + C$

$\int \csc^2(x)dx = -\cot x + C$

$\int \sec(x)\tan(x)dx = \sec x + C$

$\int \csc(x)\cot(x)dx = -\csc x + C$

Let's Practice with Trig

Find the indefinite integral

7) $\int (4\sin(x) + 6\cos(x))dx$

$-4\cos x + 6\sin x + C$

8) $\int \frac{\sin(x)}{\cos^2(x)} dx$

$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$

$\int \tan x \sec x \, dx$

$= \sec x + C$

Extra Trig Practice

Find the indefinite integral

9) $\int \frac{\tan(x)}{\cos(x)\sin(x)} dx$

$\int \frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cos x \cancel{\sin x}} dx$

$\int \frac{1}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + C$

10) $\int \frac{6\csc(x)}{\sin(x)} dx =$

$= 6 \int \frac{1}{\sin x} \cdot \frac{1}{\sin x} dx$

$6 \int \frac{1}{\sin^2 x} dx =$

$6 \int \csc^2 x \, dx$

$= -6\cot x + C$

11) $\int \frac{-4\cos(x)}{\sin^2(x)} dx$

$-4 \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$

$-4 \int \cot x \csc x \, dx = 4\csc x + C$