Name Answer Key Date________

Calc I H – 4.1 Day 3 – Integration Word Problems **Period** ________

1) A ball is thrown upward with an initial velocity of 64 ft. /sec from a height of 80 feet.
   a. Find the position function

$v(0) = 64$ $s(0) = 80$

$\int a(t)\,dt = \int -32 \frac{ft}{sec^2}\,dt$

$\int v(t)\,dt = \int -32t + 64\,dt$

$\boxed{s(t) = -16t^2 + 64t + 80}$

$v(t) = -32t + C$

$v(0) = -32(0) + C = 64$

$C = 64$

$s(t) = -16t^2 + 64t + C$

$s(0) = -16(0)^2 + 64(0) + C = 80$

$C = 80$

   b. What is the maximum height of the ball?

$\boxed{s(2) = 144 \text{ ft.}}$

$s'(t) = v(t) = -32t + 64 = 0$

$t = 2$ sec

$s'(t)$: + on (0, 2), − after 2
$s(t)$: ↗ then ↘

max occurs at $t = 2$ sec
$s(t)$ incr → decr

2) A ball is thrown vertically upwards and after 1 second the velocity is 64ft. /sec and the ball is 80 feet above ground level. Write the acceleration, velocity, and position function.

$v(1) = 64$, $s(1) = 80$

$\int a(t)\,dt = \int -32\,dt$

$v(t) = -32t + C$

$v(1) = -32(1) + C = 64$

$C = 96$

$\int v(t)\,dt = \int -32t + 96\,dt$

$s(t) = -16t^2 + 96t + C$

$s(1) = -16(1)^2 + 96(1) + C = 80$

$80 + C = 80$

$C = 0$

$$\boxed{\begin{aligned} s(t) &= -16t^2 + 96t \\ v(t) &= -32t + 96 \\ a(t) &= -32 \end{aligned}}$$

3) The rate of growth $\frac{dP}{dt}$ of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt[5]{t}$ where time is in days. The initial size of the population is 900 bacteria. After 1 day the population is 1250 bacteria. Estimate the population after 4 weeks.

$P(0) = 900$, $P(1) = 1250$ Find $P(28)$

$\int dP = \int k t^{1/5}\,dt$

$P = \frac{5k}{6} t^{6/5} + C$

$P(0) = \frac{5k}{6}(0)^{6/5} + C = 900$

$C = 900$

$P = \frac{5k}{6} t^{6/5} + 900$

$P(1) = \frac{5k}{6}(1)^{6/5} + 900 = 1250$

$\frac{5k}{6} = 350$

$\boxed{P = 350t^{6/5} + 900}$

$P(28) = 350(28)^{6/5} + 900$

$\boxed{P(28) \approx 19{,}983 \text{ bacteria}}$

4) The growth rate of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt[4]{t}$ where time is in days. The initial size of the population is 500 bacteria. After 1 day the population is 700 bacteria. Estimate the population after 3 weeks.

$P(0)=500$, $P(1)=700$ Find $P(21)$

$\int dP = \int k t^{1/4}\, dt$

$P = \frac{4k}{5} t^{5/4} + C$

$P(0) = \frac{4k}{5}(0)^{5/4} + C = 500$

$C = 500$

$P = \frac{4k}{5} t^{5/4} + 500$

$P(1) = \frac{4k}{5}(1)^{5/4} + 500 = 700$

$\frac{4k}{5} = 200$

$\boxed{P = 200\, t^{5/4} + 500}$

$P(21) = 200(21)^{5/4} + 500$

$\boxed{P(21) = 9491 \text{ bacteria}}$

5) A stone is thrown vertically upwards and after 2 seconds in flight the velocity is -24 ft. /sec and the height is 28 feet above ground level. Write the position function of the stone and then find the maximum height.

$v(2) = -24$, $s(2) = 28$

$\int a(t)\,dt = \int -32\, dt$

$v(t) = -32t + C$

$v(2) = -32(2) + C = -24$

$-64 + C = -24$

$C = 40$

$\int v(t)\,dt = \int -32t + 40\, dt$

$s(t) = -16t^2 + 40t + C$

$s(2) = -16(2)^2 + 40(2) + C = 28$

$16 + C = 28$

$C = 12$

$\boxed{s(t) = -16t^2 + 40t + 12}$

$s'(t) = -32t + 40 = 0$

$t = 1.25$

$s'(t)$: $+$ on $(0, 1.25)$, $-$ after 1.25

$s(t)$: 0 $\nearrow$ 1.25 $\searrow$

max height occurs at $t = 1.25$ since s incr $\to$ decr.

$\boxed{s(1.25) = 37 \text{ feet}}$

6) The growth rate of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt[3]{t^2}$ where time is in weeks. The initial population was 20 bacteria and increased by 15 in the first week. Estimate the population after 1 year.

$P(0)=20$, $P(1)=35$ Find $P(52)$

$\int dP = \int k t^{2/3}\, dt$

$P = \frac{3k}{5} t^{5/3} + C$

$P(0) = \frac{3k}{5}(0)^{5/3} + C = 20$

$C = 20$

$P = \frac{3k}{5} t^{5/3} + 20$

$P(1) = \frac{3k}{5}(1)^{5/3} + 20 = 35$

$\frac{3k}{5} = 15$

$P = 15\, t^{5/3} + 20$

$P(52) = 15(52)^{5/3} + 20$

$\boxed{P(52) = 10{,}887 \text{ bacteria}}$