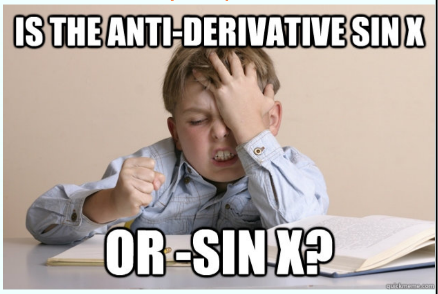
4.1 day 3 - Antiderivatives and Indefinite Integration 3/18/19



Homework:

- 4.1C
- 4.1 Quiz - Thursday, 3/21 or Friday, 3/22

Objective:

Write the general solution of a differential equation; Use indefinite integral notation for antiderivatives; Use basic integration rules to find antiderivatives.

Do Now: Find a particular solution for $\int -32\,dx$ given $f(1) = -22$

Then find a general solution for $\int -32x + 10\,dx$.

Do Now

Find a particular solution for $\int -32\,dx$ given $f(1) = -22$.

Gen soln $\Rightarrow$ $f(x) = -32x + C$ $f(x) = -32x + 10$

$f(1) = -22 = -32(1) + C$

$10 = C$

Then find a general solution for $\int -32x + 10\,dx$.

$g(x) = -16x^2 + 10x + C$

HW Questions???

$f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

$\int f''(x)\,dx = \int \sin x\,dx$

$f'(x) = -\cos x + C$

$f'(0) = -\cos(0) + C = 1$

$-1 + C = 1$

$C = 2$

$\int f'(x)\,dx = \int -\cos x + 2\,dx$

$f(x) = -\sin x + 2x + C$

$f(0) = -\sin(0) + 0 + C = 6$

$C = 6$

$\boxed{f(x) = -\sin x + 2x + 6}$

1) A ball is thrown upward with an initial velocity of 64 ft./sec. from a height of 80 feet.

a. Find the position function.

$\int a(t)\,dt = \int -32\,dt$

$V(t) = -32t + C$

$V(0) = 64 = -32(0) + C$

$C = 64$

$\int V(t)\,dt = \int -32t + 64\,dt$

$S(t) = -16t^2 + 64t + C$

$S(0) = 80 = -16(0) + 64(0) + C$

$C = 80$

$\boxed{S(t) = -16t^2 + 64t + 80}$

b. What is the maximum height of the ball?

$V(t) = 0 = -32t + 64$

$t = 2$ secs.

$S(2) = -16(2)^2 + 64(2) + 80$

$= 144$ ft

2) A ball is thrown vertically upwards and after 1 second the velocity is 64 ft./sec. and the ball is 80 ft above ground level. Write the acceleration, velocity, and position function.

$\int a(t)\,dt = \int -32\,dt$

$V(t) = -32t + C$

$V(1) = 64 = -32(1) + C$

$C = 96$

$\boxed{V(t) = -32t + 96}$

$\int V(t)\,dt = \int -32t + 96\,dt$

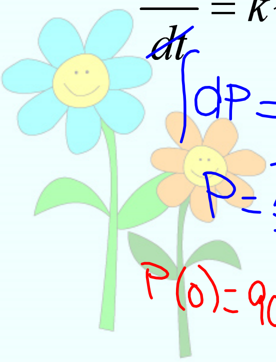
$S(t) = -16t^2 + 96t + C$

$S(1) = 80 = -16(1)^2 + 96(1) + C$

$C = 0$

$\boxed{S(t) = -16t^2 + 96t}$

3) The rate of growth $\frac{dP}{dt}$ of a population of bacteria is given by where time is in days. The initial size of the population is 900 bacteria. After 1 day the population is 1250 bacteria. Estimate the population after 4 weeks.



$$\frac{dP}{\cancel{dt}} = k\sqrt[5]{t}\ \cancel{dt}$$

$$\int dP = \int kt^{\frac{1}{5}}\,dt$$

$$P = \frac{5kt^{\frac{6}{5}}}{6} + C$$

$$P(0) = 900 = \frac{5k}{6}(0) + C$$

$$C = 900$$

$$P = \frac{5k}{6}t^{\frac{6}{5}} + 900$$

$$P(1) = 1250 = \frac{5k}{6}(1) + 900$$

$$\frac{6}{5}\cdot 350 = \frac{5k}{6}\cdot\frac{6}{5}$$

$$420 = k$$

Skip

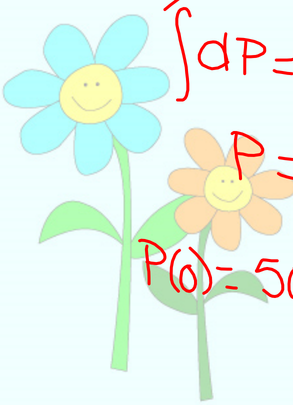
$$P = \frac{5}{6}(420)t^{\frac{6}{5}} + 900$$

$$P = 350t^{\frac{6}{5}} + 900$$

$$P(28) = 19{,}983.485$$

$$= 19{,}983 \text{ bact.}$$

4) The growth rate of a population of bacteria is given by where time is in days. The initial size of the population is 500 bacteria. After 1 day the population is 700 bacteria. Estimate the population after 3 weeks.



$$\frac{dP}{\cancel{dt}} = k\sqrt[4]{t}\ \cancel{dt}$$

$$\int dP = \int kt^{\frac{1}{4}}\,dt$$

$$P = \frac{4}{5}kt^{\frac{5}{4}} + C$$

$$P(0) = 500 = \frac{4}{5}k(0) + C$$

$$C = 500$$

$$P = \frac{4}{5}kt^{\frac{5}{4}} + 500$$

$$P(1) = 700 = \frac{4}{5}k + 500$$

$$200 = \frac{4}{5}k$$

$$P = 200t^{5/4} + 500$$

$$P(21) = 9490.919$$

9490 or 9491 bact.

5) A stone is thrown vertically upwards and after 2 seconds in flight the velocity is -24 ft. /sec and the height is 28 feet above ground level. Write the position function of the stone and then find the maximum height.

$V(t) = -32t + 40$

$S(t) = -16t^2 + 40t + 12$

$S(1.25) = 37$ ft

6) The growth rate of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt[3]{t^2}$ where time is in weeks. The initial population was 20 bacteria and increased by 15 in the first week. Estimate the population after 1 year.

$P(t) = 15t^{5/3} + 20$

$P(52) = 10{,}887$ bact.

Bonus

1) A stone is launched and after 1 second the stone is 120 feet in the air and its velocity is 32 ft/sec. Write the position function for this stone.

2) The growth rate of a population of bacteria is given by $\frac{dP}{dt} = k\sqrt[5]{t^2}$ where time is in days. The initial size of the population is 100 bacteria. After 1 day the population is 700 bacteria. Estimate the population after 2 weeks.