Name Answer key

Calc I H – 4.2 & 4.6 – Day 1 Riemann Sums

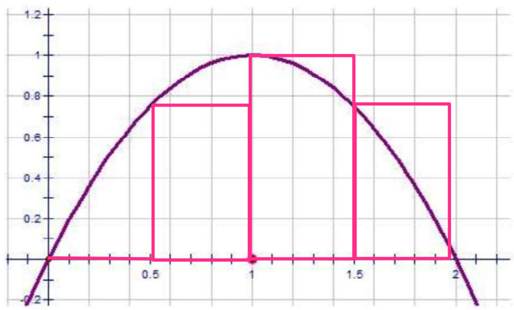
Date

Period

1. The region R is enclosed between the function $f(x)=2x-x^2$ and the $x-axis$ for $0 \le x \le 2$.

On the graphs below, draw the LRAM, RRAM and MRAM using 4 equal subintervals that would approximate the area of R. Then, set-up the formula and find the area using each method.

LRAM



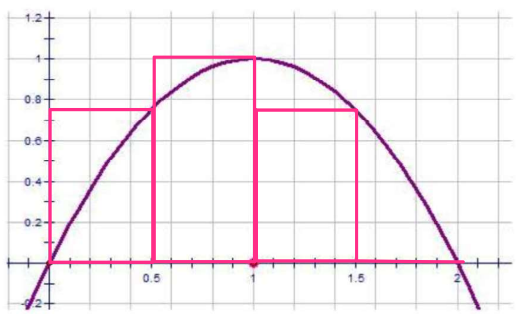
$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

$$A = \Delta x\left(f(0) + f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right)\right)$$

$$= \frac{1}{2}\left(0 + \frac{3}{4} + 1 + \frac{3}{4}\right) = \frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}\,u^2$$

$$= 1.25u^2$$

RRAM

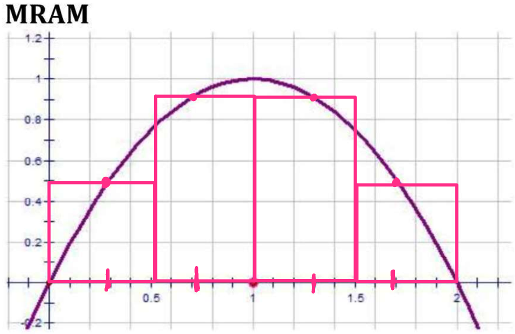


$$A = \Delta x\left(f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right) + f(2)\right)$$

$$= \frac{1}{2}\left(\frac{3}{4} + 1 + \frac{3}{4} + 0\right) = \frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}\,u^2$$

$$= 1.25u^2$$

MRAM



$$A = \Delta x\left(f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right)\right)$$

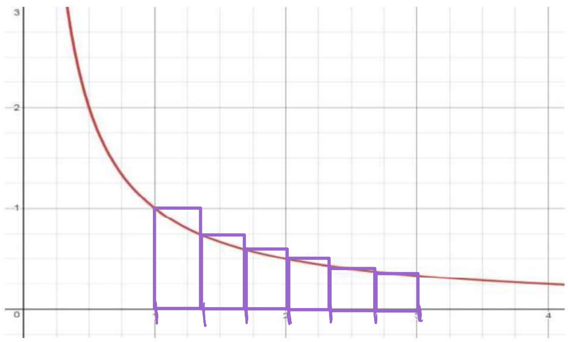
$$= \frac{1}{2}\left(\frac{7}{16} + \frac{15}{16} + \frac{15}{16} + \frac{7}{16}\right)$$

$$= \frac{1}{2}\left(\frac{44}{16}\right) = \frac{11}{8} = 1.375\,u^2$$

2. The region R is enclosed between the function $h(x)=\frac{1}{x}$ and the $x-axis$ for $1 \le x \le 3$.

On the graphs below, draw the LRAM, RRAM and MRAM using 6 equal subintervals that would approximate the area of R. Then, set-up the formula and find the area using each method.

LRAM



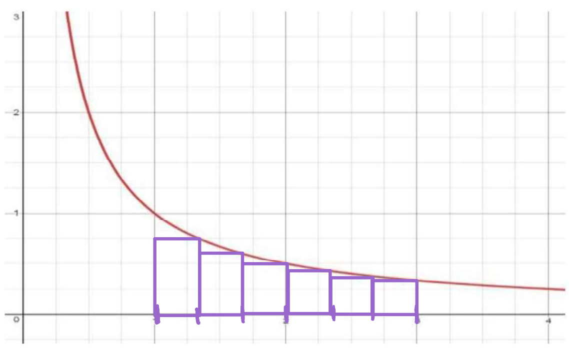
$\Delta x = \frac{b-a}{n} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$

$A = \Delta x \left(f(1) + f(\frac{4}{3}) + f(\frac{5}{3}) + f(2) + f(\frac{7}{3}) + f(\frac{8}{3}) \right)$

$= \frac{1}{3}\left(1 + \frac{3}{4} + \frac{3}{5} + \frac{1}{2} + \frac{3}{7} + \frac{3}{8}\right)$

$\approx 1.218\ u^2$

RRAM



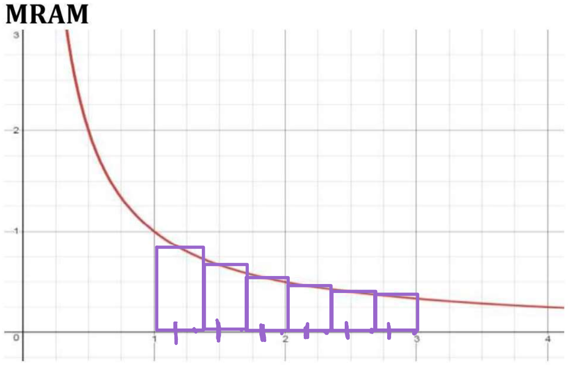
$A = \Delta x \left(f(\frac{4}{3}) + f(\frac{5}{3}) + f(2) + f(\frac{7}{3}) + f(\frac{8}{3}) + f(3) \right)$

$= \frac{1}{3}\left(\frac{3}{4} + \frac{3}{5} + \frac{1}{2} + \frac{3}{7} + \frac{3}{8} + \frac{1}{3}\right)$

$\approx .996\ u^2$

MRAM

$\frac{1+\frac{4}{3}}{2} = \frac{\frac{7}{3}}{2} = \frac{7}{6}$



$A = \Delta x \left(f(\frac{7}{6}) + f(\frac{3}{2}) + f(\frac{11}{6}) + f(\frac{13}{6}) + f(\frac{5}{2}) + f(\frac{17}{6}) \right)$

$= \frac{1}{3}\left(\frac{6}{7} + \frac{2}{3} + \frac{6}{11} + \frac{6}{13} + \frac{2}{5} + \frac{6}{17}\right)$

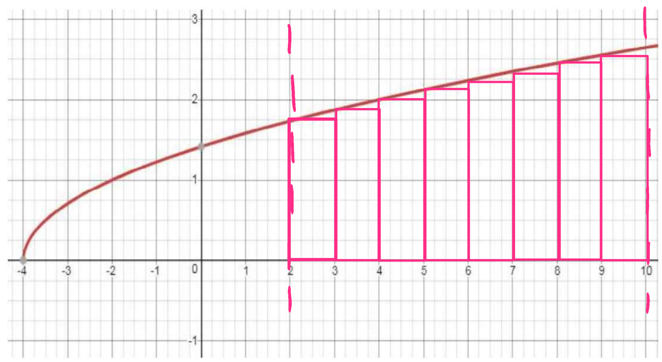
$\approx 1.095\ u^2$

Homework: The region R is enclosed between the function $g(x)=\sqrt{\frac{x}{2}+2}$ and the $x-axis$ for $2\le x\le 10$.

On the graphs below, draw the LRAM, RRAM and MRAM using 8 equal subintervals that would approximate the area of R. Then, set-up the formula and find the area using each method.

LRAM

$\Delta x = \frac{b-a}{n} = \frac{10-2}{8} = 1$

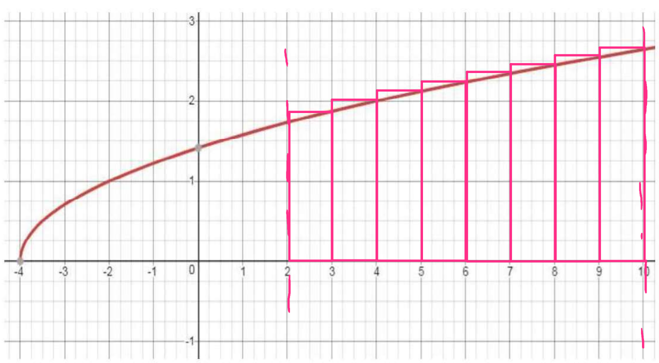


$A = \Delta x\left(f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)\right)$

$= 1\left(\sqrt{3}+\sqrt{\frac{7}{2}}+2+\sqrt{\frac{9}{2}}+\sqrt{5}+\sqrt{\frac{11}{2}}+\sqrt{6}+\sqrt{\frac{13}{2}}\right)$

$\approx 17.304\ u^2$

RRAM



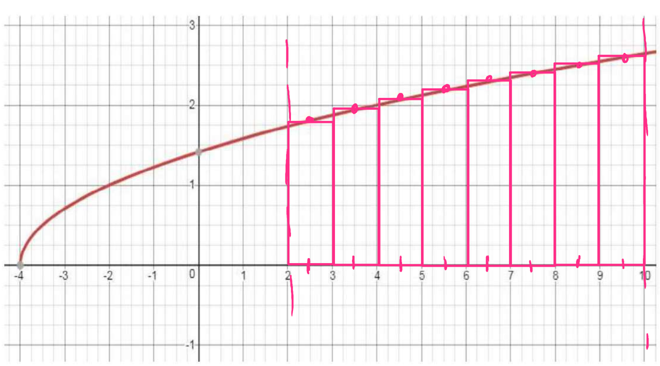
$A = \Delta x\left(f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)+f(10)\right)$

$= 1\left(\sqrt{\frac{7}{2}}+2+\sqrt{\frac{9}{2}}+\sqrt{5}+\sqrt{\frac{11}{2}}+\sqrt{6}+\sqrt{\frac{13}{2}}+\sqrt{7}\right)$

$\approx 18.218\ u^2$

MRAM

$f(x)=\sqrt{\frac{x}{2}+2}$



$A = \Delta x\left(f\left(\frac{5}{2}\right)+f\left(\frac{7}{2}\right)+f\left(\frac{9}{2}\right)+f\left(\frac{11}{2}\right)+f\left(\frac{13}{2}\right)+f\left(\frac{15}{2}\right)+f\left(\frac{17}{2}\right)+f\left(\frac{19}{2}\right)\right)$

$= 1\left(\sqrt{\frac{13}{4}}+\sqrt{\frac{15}{4}}+\sqrt{\frac{17}{4}}+\sqrt{\frac{19}{4}}+\sqrt{\frac{21}{4}}+\sqrt{\frac{23}{4}}+\sqrt{\frac{25}{4}}+\sqrt{\frac{27}{4}}\right)$

$\approx 17.768\ u^2$