Class Notes:

Definition of Definite Integral: (Refer to www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/definite_integrals/v/riemann-sums-and-integrals)

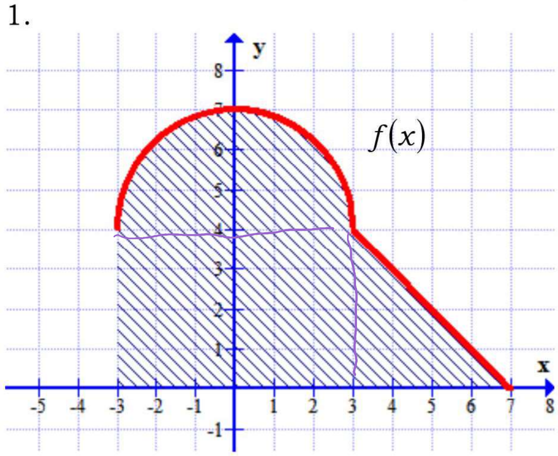
Example using LRAM:

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_{i-1})\Delta x \text{, where } \Delta x = \frac{b-a}{n}$$

Example:

Find the area of each region below geometrically. Set up an integral and evaluate each area.

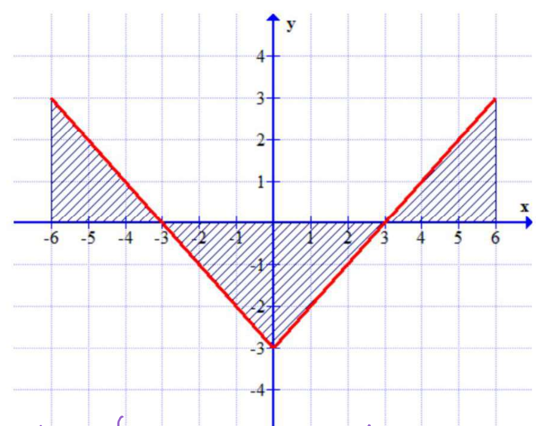
1.



$A = 6(4) + \frac{1}{2}(4)(4) + \frac{\pi}{2}(3)^2$

$A = 24 + 8 + \frac{9\pi}{2} = 32 + \frac{9\pi}{2}$

2.



$A = \frac{1}{2}(3)(3) - \frac{1}{2}(6)(3) + \frac{1}{2}(3)(3) = 0$

Continuity Implies Integrability:

If a function f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

The Definite Integral as the Area of a Region:

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by:

$$\text{Area} = \int_a^b f(x)\,dx.$$

If f is continuous and negative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by:

$$\text{Area} = -\int_a^b f(x)\,dx.$$

For any integrable function f, $\int_a^b f(x)\,dx = (\text{area above the } x\text{ - axis}) - (\text{area below the } x\text{ - axis})$

Properties of Definite Integrals

1. ***Order of Integration:*** $\int_b^a f(x)dx = -\int_a^b f(x)dx$

2. ***Zeros:*** $\int_a^a f(x)dx = 0$

3. ***Constant Multiple:*** $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for *any number k*

4. ***Sum and Difference:*** $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5. ***Additivity:*** $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

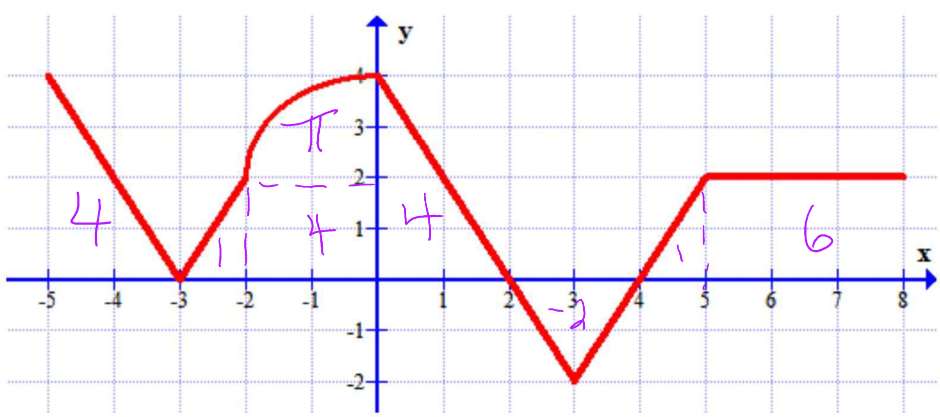
6. ***Domination:*** $f(x) \geq g(x)$ **on [a, b]** $\Rightarrow \int_a^b f(x)dx \geq \int_a^b g(x)dx$

   $f(x) \geq 0$ on [a, b] $\Rightarrow \int_a^b f(x)dx \geq 0$

Examples:

1. The graph of *f(x)* consists of line segments and a quarter circle. Use geometric formulas to evaluate the definite integrals.

$\frac{\pi}{4}(2)^2$



a. $\int_{-5}^{-2} f(x)dx = 4+1 = \boxed{5}$

b. $\int_{-2}^{2} f(x)dx = \boxed{8+\pi}$ $4+\pi+4$

c. $\int_{2}^{5} f(x)dx = -2+1 = \boxed{-1}$

d. $\int_{5}^{8} f(x)dx = \boxed{6}$

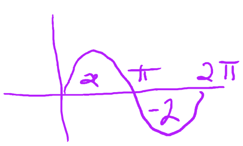
e. $\int_{-5}^{8} |f(x)|dx = \boxed{22+\pi}$ $4+1+4+\pi+4+2+1+6$

f. $\int_{-5}^{8} [f(x)+3]dx$ $= \int_{-5}^{8} f(x)dx + \int_{-5}^{8} 3dx = 18+\pi+13(3) = \boxed{57+\pi}$

2. Suppose $\int_{-2}^{5} f(x)dx=18$, $\int_{-2}^{5} g(x)dx=5$, $\int_{-2}^{5} h(x)dx=-11$, and $\int_{-2}^{8} f(x)dx=0$, find:

a) $\int_{-2}^{5}(f(x)+g(x))dx$ $=18+5=\boxed{23}$	**b)** $\int_{-2}^{5}[f(x)+g(x)-h(x)]dx$ $=18+5-(-11)$ $=\boxed{34}$	**c)** $\int_{5}^{-2}4g(x)dx$ $=-4\int_{-2}^{5}g(x)dx$ $=-4(5)=\boxed{-20}$
d) $\int_{-2}^{5}(g(x)+2)dx$ $=\int_{-2}^{5}g(x)dx+\int_{-2}^{5}2dx$ $=5+7(2)=\boxed{19}$	e) $\int_{-2}^{5}(f(x)-6)dx$ $=\int_{-2}^{5}f(x)dx-\int_{-2}^{5}6dx$ $=18-7(6)=\boxed{-24}$	f) * $\int_{0}^{7}h(x-2)dx=\int_{-2}^{5}h(x)dx$ $=\boxed{-11}$ *Undo translation → 2
g) * $\int_{-4}^{3}g(x+2)dx$ $=\int_{-2}^{5}g(x)dx=\boxed{5}$ *undo translation ← 2	h) $\int_{5}^{8}f(x)dx$ $=\int_{-2}^{8}f(x)dx-\int_{-2}^{5}f(x)dx$ $=0-18=\boxed{-18}$	i) $\int_{1}^{8}[f(x-3)+3]dx$ $=\int_{-2}^{5}f(x)dx+\int_{1}^{8}3dx$ $=18+7(3)=\boxed{39}$

3. Evaluate $\int_0^{\pi}sin(x)\,dx$ $=2$ using the calculator. Use this fact to answer the following:

a) $\int_{\pi}^{2\pi}sin(x)\,dx=\boxed{-2}$ 	b) $\int_{0}^{2\pi}sin(x)\,dx=\boxed{0}$	c) $\int_{0}^{\frac{\pi}{2}}sin(x)\,dx=\boxed{1}$
d) $\int_{0}^{\pi}(2+sin(x))\,dx$ $\int_0^{\pi}2dx+\int_0^{\pi}\sin x\,dx$ $\boxed{2\pi+2}$	e) $\int_{0}^{\pi}2\,sin(x)\,dx=2\int_0^{\pi}\sin x\,dx$ $=2(2)=\boxed{4}$	f) $\int_{2}^{\pi+2}sin(x-2)\,dx$ →2 $=\int_0^{\pi}\sin x\,dx=\boxed{2}$
g) $\int_{0}^{\pi}-\sin(u)\,du$ $=-\int_0^{\pi}\sin u\,du$ $=\boxed{-2}$	h) $\int_{0}^{2\pi}sin\left(\frac{x}{2}\right)dx$ Per $=\frac{2\pi}{\frac{1}{2}}=4\pi$ $=2\int_0^{\pi}\sin x\,dx=\boxed{4}$	i) $\int_{0}^{\pi}cos(x)\,dx=\boxed{0}$