4.3 - Definite Integrals

1/16/19

$$life = \int (choice)\, dt$$

Homework:

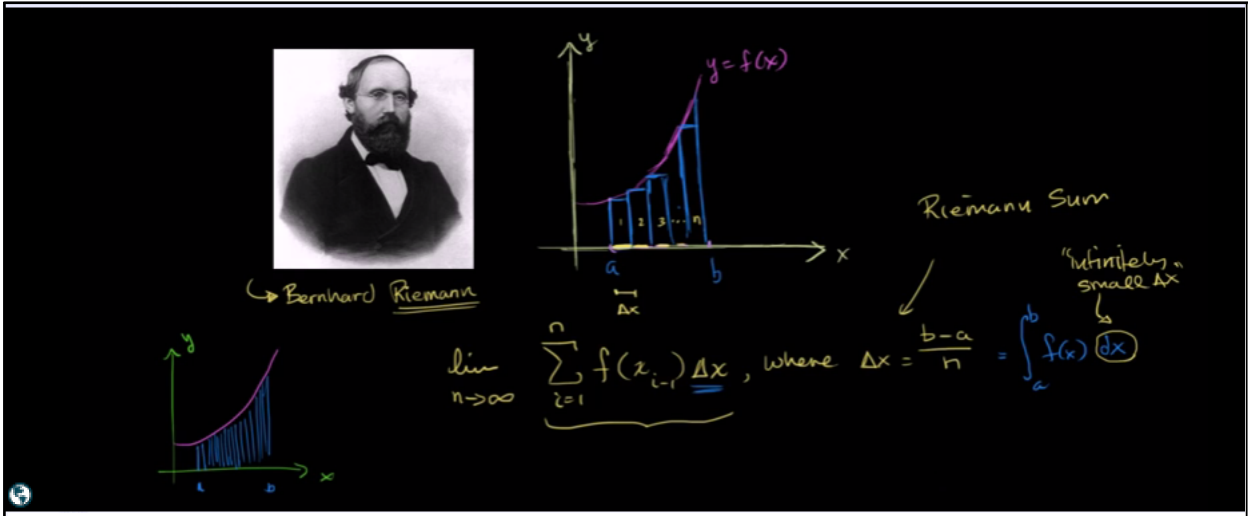
- Section 4.3 A
- Complete 4.3 Worksheet
- 4.2-4.3, 4.6 & Linear Approx PA - Friday, 1/18

Objective:

Understand the definite integral as the area under the curve; Evaluate definite integrals using geometric formulas; Evaluate definite integrals using properties.

Do Now:

Take a handout from the bin...



Definition of Definite Integral: (Refer to www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/definite_integrals/v/riemann-sums-and-integrals)

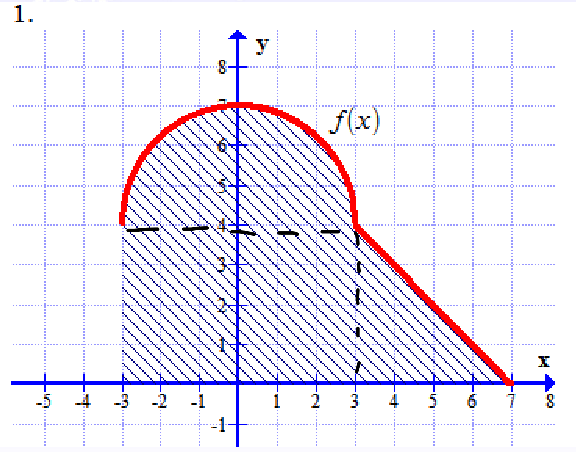
Example using LRAM:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\Delta x \text{, where } \Delta x = \frac{b-a}{n}$$

Classwork

Find the area of each region below geometrically. Set up an integral and evaluate each area.

1.

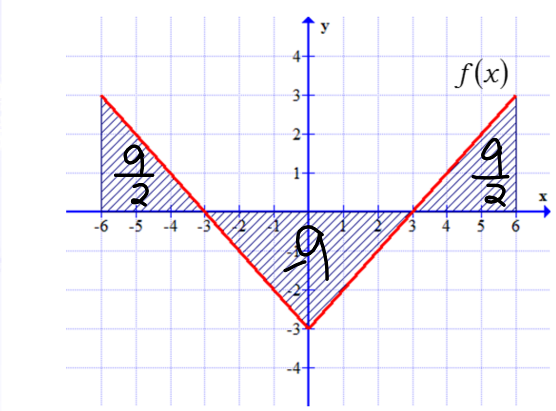


$$\int_{-3}^{7} f(x)\,dx =$$

$$32+\frac{9\pi}{2}$$

Find the area of each region below geometrically. Set up an integral and evaluate each area.

2.



$$\int_{-6}^{6} f(x)\,dx =$$

$$\frac{9}{2}-9+\frac{9}{2}=0$$

Homework questions?

Continuity Implies Integrability:
If a function f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

How does this compare to differentiability?
Does continuity imply differentiability?

The Definite Integral as the Area of a Region:
If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by:

$$\text{Area} = \int_a^b f(x)\,dx.$$

If f is continuous and negative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by:

$$\text{Area} = -\int_a^b f(x)\,dx.$$

For any integrable function f, $\int_a^b f(x)\,dx = (\text{area above the } x\text{ - axis}) - (\text{area below the } x\text{ - axis})$

Properties of Definite Integrals

1. **Order of Integration:** $\int_b^a f(x)dx = -\int_a^b f(x)dx$

2. **Zeros:** $\int_a^a f(x)dx = 0$

3. **Constant Multiple:** $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any number k

4. **Sum and Difference:** $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

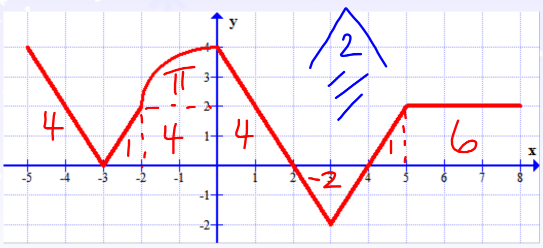
5. **Additivity:** $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

6. **Domination:** $f(x) \geq g(x)$ **on [a, b]** $\Rightarrow \int_a^b f(x)dx \geq \int_a^b g(x)dx$

$f(x) \geq 0$ on [a, b] $\Rightarrow \int_a^b f(x)dx \geq 0$

Examples

1. The graph of *f(x)* consists of line segments and a quarter circle. Use geometric formulas to evaluate the definite integrals



a. $\int_{-5}^{-2} f(x)dx = 5$

b. $\int_{-2}^{2} f(x)dx = 8+\pi$

c. $\int_{2}^{5} f(x)dx$

$= -1$

d. $\int_{5}^{8} f(x)dx = 6$

$\left|\int_{-5}^{8} f(x)dx\right|$

e. $\int_{-5}^{8} |f(x)|dx$

$\boxed{22+\pi}$

$18 \cancel{+\pi}$

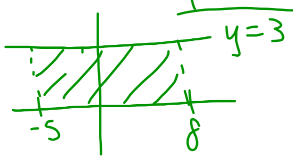
$\cancel{20+\pi}$

f. $\int_{-5}^{8} [f(x)+3]dx$

$\int_{-5}^{8} f(x)dx + \int_{-5}^{8} 3dx$

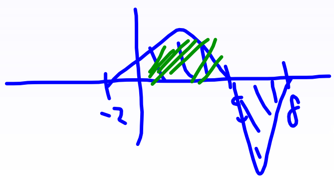
$18+\pi+13(3)$

$\boxed{57+\pi}$



2. Suppose $\int_{-2}^{5} f(x)dx = 18$, $\int_{-2}^{5} g(x)dx = 5$, $\int_{-2}^{5} h(x)dx = -11$, and $\int_{-2}^{8} f(x)dx = 0$, find:

a) $\int_{-2}^{5}(f(x)+g(x))dx$ $18+5=23$	b) $\int_{-2}^{5}[f(x)+g(x)-h(x)]\,dx$ $18+5-(-11)=34$	c) $\int_{5}^{-2}4g(x)dx$ $-4\int_{-2}^{5}g(x)dx = -4(5)=-20$
d) $\int_{-2}^{5}(g(x)+2)dx$ $\int_{-2}^{5}g(x)dx+\int_{-2}^{5}2dx$ $5+7(2)=19$	e) $\int_{-2}^{5}(f(x)-6)dx$ $\int_{-2}^{5}f(x)dx-\int_{-2}^{5}6dx$ $= 18-7(6)=-24$	f) $\int_{0}^{7}h(x-2)dx$ $\int_{-2}^{5}h(x)dx = -11$
g) $\int_{-4}^{3}g(x+2)dx$ $\int_{-2}^{5}g(x)dx=5$	h) $\int_{5}^{8}f(x)dx$ $\int_{-2}^{8}f(x)dx-\int_{-2}^{5}f(x)dx$ $0-18=-18$	i) $\int_{1}^{8}[f(x-3)+3]\,dx$ $\int_{-2}^{5}f(x)dx+\int_{1}^{8}3dx$ $18+21=39$



3. Given $\int_0^{\pi} \sin(x)\,dx = 2$, evaluate the following:

a) $\int_{\pi}^{2\pi} sin(x)dx$	b) $\int_{0}^{2\pi} sin(x)dx$	c) $\int_{0}^{\frac{\pi}{2}} sin(x)dx$
d) $\int_{0}^{\pi}(2+sin(x))dx$	e) $\int_{0}^{\pi}2\,sin(x)dx$	f) $\int_{2}^{\pi+2} sin(x-2)dx$
g) $\int_{0}^{\pi}-\sin(u)\,du$	h) $\int_{0}^{2\pi} sin\left(\frac{x}{2}\right)dx$	i) $\int_{0}^{\pi} cos(x)dx$