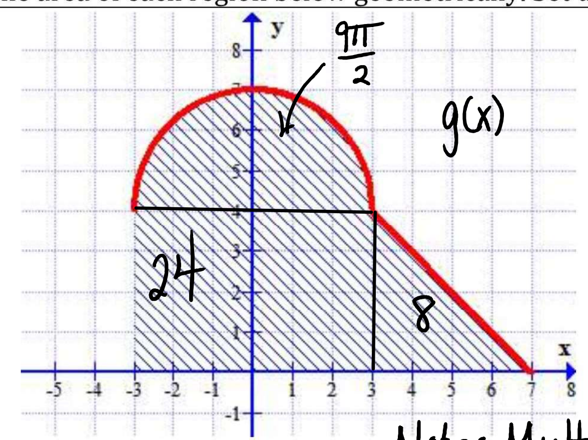
Name Answer Key Date

Calc I H – 4.3 day 1 – Intro Period

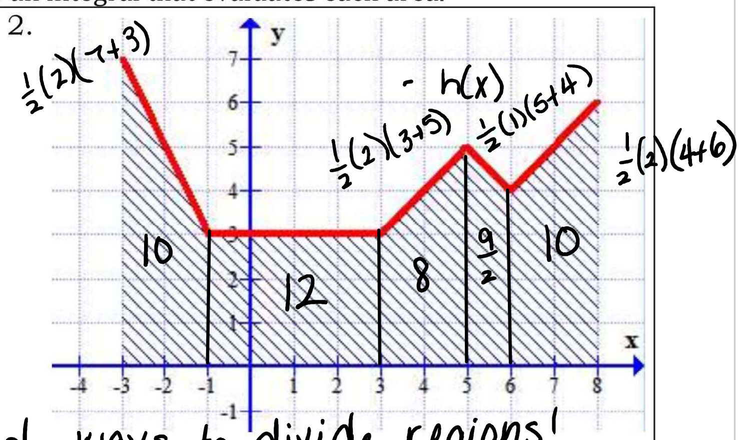
Do Now:

Find the area of each region below geometrically. Set up an integral that evaluates each area.

1.



2.



Note: Multiple ways to divide regions!

$$\int_{-3}^{7} g(x)\,dx = 32 + \frac{9\pi}{2}$$

$$\int_{-3}^{8} h(x)\,dx = 10 + 12 + 8 + \frac{9}{2} + 10$$
$$= 44.5$$

The Definite Integral as the Area of a Region:

If f is continuous and ***positive*** on the closed interval $[a,\ b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x=a$ and $x=b$ is given by:

$$\text{Area} = \int_a^b f(x)\,dx.$$

If f is continuous and ***negative*** on the closed interval $[a,\ b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x=a$ and $x=b$ is given by:

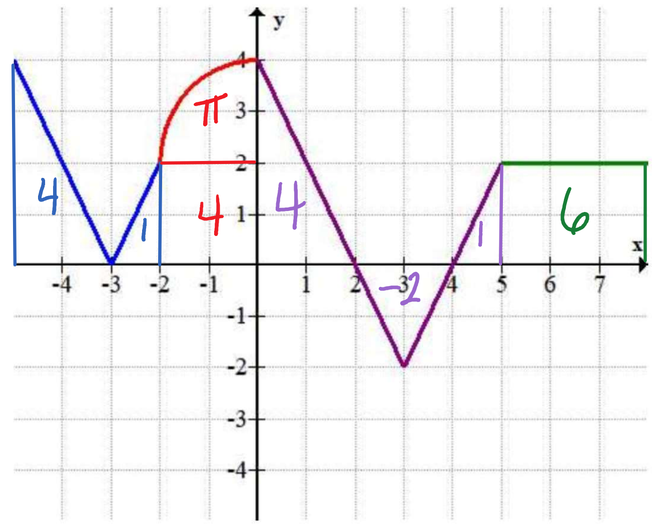
$$\text{Area} = -\int_a^b f(x)\,dx.$$

For any integrable function f,

$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis})$$

Examples:

The graph below consists of line segments and a quarter-circle. Evaluate each definite integral by using geometric formulas.



1. $\int_{-5}^{-2} f(x)dx =$ 5

2. $\int_{-2}^{2} f(x)dx =$ 8+π

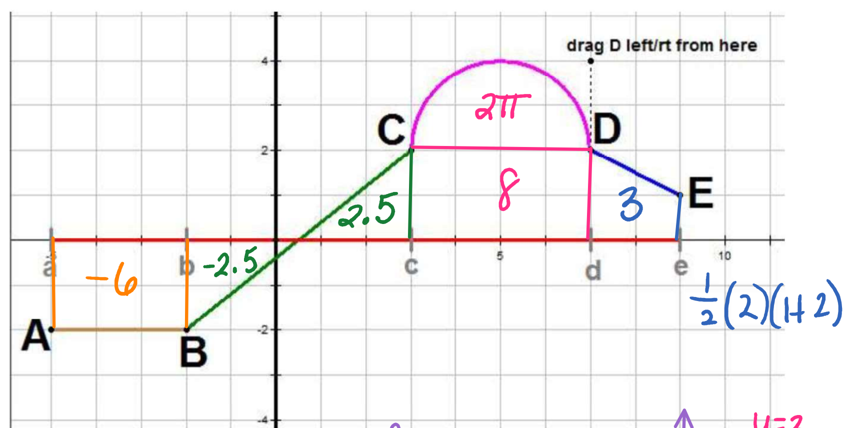
3. $\int_{2}^{4} f(x)dx =$ −2

4. $\int_{4}^{5} f(x)dx =$ 1

5. $\int_{5}^{8} f(x)dx =$ 6

6. $\int_{-5}^{8} f(x)dx =$ 18+π

The graph of f consists of line segments and a semicircle, as shown below. Evaluate each definite integral by using geometric formulas.



7. $\int_{-5}^{-2} f(x)\ dx$ = −6

8. $\int_{-2}^{3} f(x)\ dx$ = 0

9. $\int_{3}^{7} f(x)\ dx$ = 8+2π

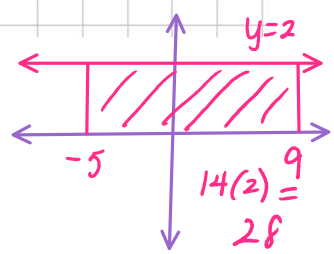
10. $\int_{7}^{9} f(x)\ dx$ = 3

11. $\int_{-5}^{9} f(x)\ dx$ = 5+2π

$= \int_{-5}^{9} f(x)dx + \int_{-5}^{9} 2dx$

$= 5+2\pi + 28$

$= 2\pi+33$

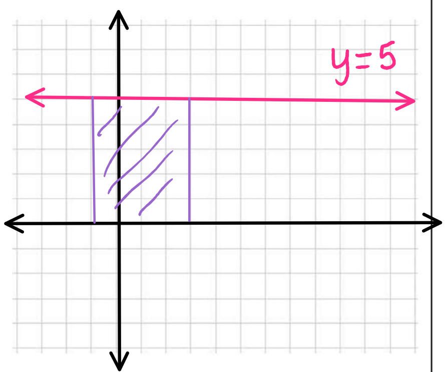


12. $\int_{-5}^{9} [f(x)+2]\ dx$

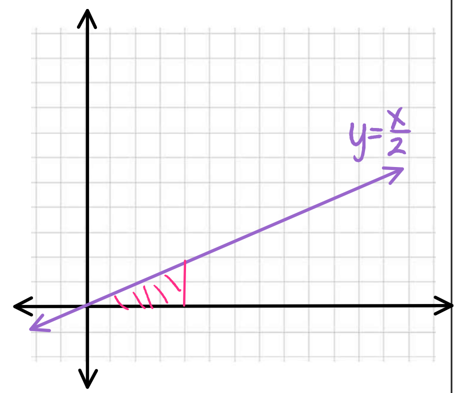
(Hint: think about how the graph $f(x)+2$ is transformed from the graph of $f(x)$)

Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

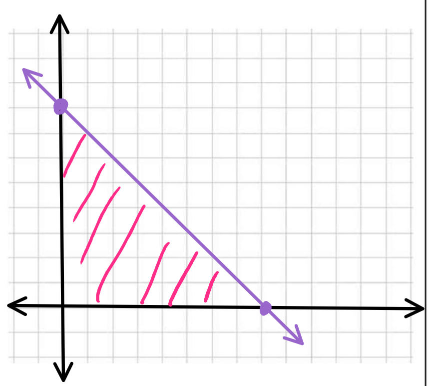
13. $\int_{-1}^{3} 5\,dx$ $= 4(5) = 20$



14. $\int_{0}^{4} \frac{x}{2}\,dx$ $= \frac{1}{2}(4)(2) = 4$



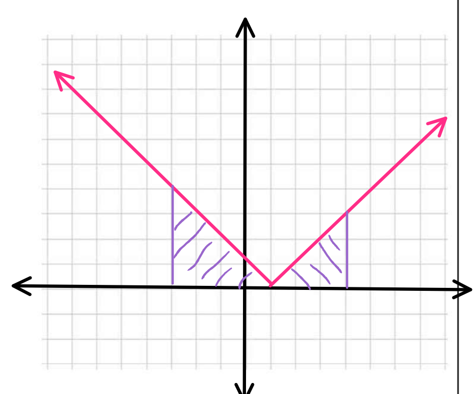
15. $\int_{0}^{8} (8-x)\,dx$ $= \frac{1}{2}(8)(8) = 32$



16. $\int_{-3}^{4} |x-1|\,dx$ $=$

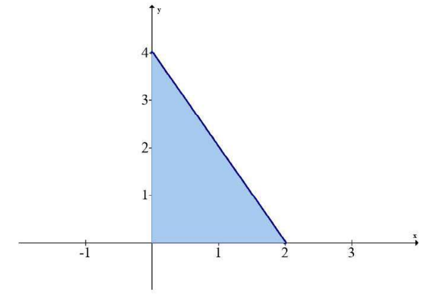
$\frac{1}{2}(4)(4) + \frac{1}{2}(3)(3)$

$= 8 + \frac{9}{2} = 12.5$



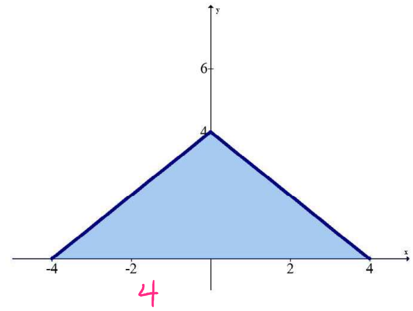
<u>Set up</u> a definite integral that yields the area of the region. (Do NOT evaluate the integral).

17. $f(x) = 4 - 2x$



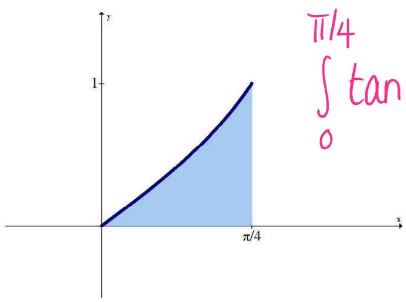
$$\int_0^2 (4-2x)\,dy$$

18. $f(x) = 4 - |x|$



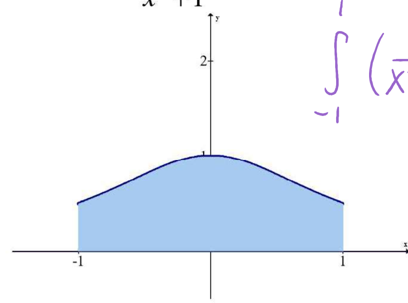
$$\int_{-4}^{4} (4-|x|)\,dx$$

19. $f(x) = \tan x$



$$\int_0^{\pi/4} \tan x\,dx$$

20. $f(x) = \dfrac{1}{x^2+1}$



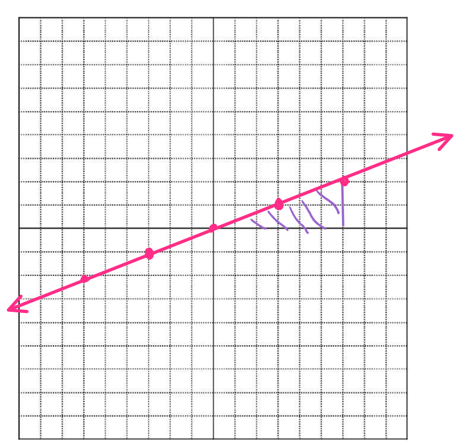
$$\int_{-1}^{1} \left(\frac{1}{x^2+1}\right) dx$$

Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral ($a > 0, r > 0$).

21. $\int_0^6 \frac{x}{3}\,dx$

$= \frac{1}{2}(6)(2)$

$= 6$



22. $\int_0^2 (x+1)\,dx$

$= \frac{1}{2}(2)(1+3)$

$= 4$