4.4 day 2 - Fundamental Thm of Calc

2/4/19

Homework:

- Section 4.4 B
- 4.1 & 4.4 Quiz - Monday, 2/11/18

Objective:

Understand and use the Second Fundamental Thm of Calc and the Thm for Integrals; Find the average of a function over a closed inte

Do Now:

For #1 and 2, evaluate the definite integral and use a graphing utility to verify your result.

1) $\int_1^8 \frac{x-x^2}{2\sqrt[3]{x}}\,dx = -\frac{3081}{80}$

$= -38.5125$

2) $\int_{\pi/4}^{\pi/2}\left(2-\csc^2 x\right)dx = \frac{\pi}{2} - 1$

Do Now

For #3, evaluate the definite integral in terms of x. Then, take the derivative

3) $F(x) = \int_{\pi/2}^{x^3}(\cos t)\,dt$

$F(x) = \sin t \Big|_{\pi/2}^{x^3}$

$F(x) = \sin(x^3) - 1$

$F'(x) = 3x^2\cos(x^3)$

The Second Fundamental Theorem of Calculus

If f is continuous on $[a,b]$, then the function $F(x)=\int_a^x f(t)dt$ has a derivative at every point x in $[a,b]$, and $\frac{dF}{dx}=\frac{d}{dx}\int_a^x f(t)dt=f(x)$.

This equation says that every continuous function is the derivative of some other function, and that every continuous function has an antiderivative.

The processes of integration and differentiation are inverses of each other.

$$\frac{dF}{dx}=\frac{d}{dx}\int_a^{g(x)} f(t)dt=f(g(x))g'(x)$$

Example 1

$$\boxed{\frac{dF}{dx}=\frac{d}{dx}\int_a^{g(x)} f(t)dt=f(g(x))g'(x)}$$

a) $\frac{d}{\cancel{dx}}\cancel{\int_0^x}\sqrt{t^2+1}\,dt$

$=\sqrt{x^2+1}$

b) $\frac{d}{\cancel{dx}}\cancel{\int_{\pi/2}^{\sqrt{x}}}(\sin t)\,dt$

$=\sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$

$$\boxed{\frac{\sin(\sqrt{x})}{2\sqrt{x}}}$$

Example 1

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^{g(x)} f(t)dt = f\big(g(x)\big)g'(x)$$

c) Find $F'(x)$.

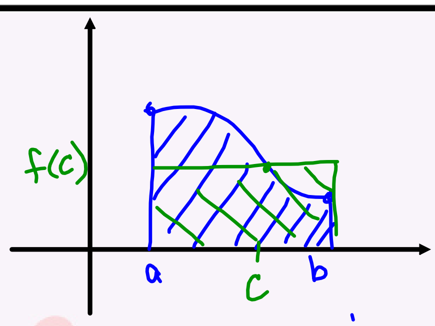
$$F(x) = \int_2^{\sin x}\left(\sqrt{t} + \cos t\right)dt$$

$$F'(x) = \left[\sqrt{\sin x} + \cos(\sin x)\right]\cos x$$

If f is integrable on $[a,b]$, its **Average (Mean) Value** on $[a,b]$ is $av(f) = \frac{1}{b-a}\int_a^b f(x)dx$.

The Mean Value Theorem for Definite Integrals

If f is continuous on $[a,b]$, then at some point c in $[a,b]$, $f(c) = av(f) = \frac{1}{b-a}\int_a^b f(x)dx$.



The Graphical Interpretation of the Mean Value Theorem for Definite Integrals:

If f is continuous on $[a,b]$, then at some point c in $[a,b]$ there is a rectangle with height $f(c)$, and length $b-a$, such that the area of the rectangle equals the area under the curve $f(x)$ on the interval $[a,b]$.

Example 2

Find the average value of the function on the interval. At what point(s) in the interval does the function assume its average value?

a) $f(x) = -\frac{x^2}{2}$, $[0, 3]$

$$\text{Av } f = \frac{1}{3-0}\int_0^3 \left(-\frac{x^2}{2}\right)dx$$

$$= \frac{1}{3}\left[-\frac{1}{6}x^3\right]_0^3 = \frac{1}{3}\left[-\frac{1}{6}(27) - 0\right] = \frac{1}{3}\left(-\frac{9}{2}\right) = \frac{-3}{2}$$

$$f(c) = \frac{-c^2}{2} = \frac{-3}{2} \qquad c = \pm\sqrt{3}$$

$$c^2 = 3 \qquad \boxed{c = \sqrt{3}}$$

b) $y = (x-1)^2$, $[0, 3]$

$$\text{Av } y = \frac{1}{3-0}\int_0^3 (x^2 - 2x + 1)dx$$

$$= \frac{1}{3}\left(\frac{x^3}{3} - x^2 + x\right)_0^3 = \frac{1}{3}(9 - 9 + 3 - 0) = 1$$

$$f(c) = (c-1)^2 = 1$$

$$c - 1 = \pm 1$$

$$c = 1 \pm 1$$

$$\boxed{c = 2, 0}$$

c) $f(x)=\sec(x)\tan(x)$, $\left[0,\frac{\pi}{3}\right]$

$$\text{Av}f=\frac{1}{\frac{\pi}{3}-0}\int_0^{\pi/3}\sec x\tan x\,dx$$

$$=\frac{3}{\pi}\Big[\sec x\Big]_0^{\pi/3}=\frac{3}{\pi}\left[\sec\tfrac{\pi}{3}-\sec 0\right]$$

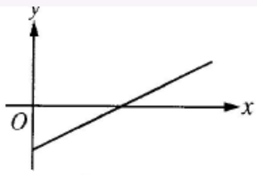
$$=\frac{3}{\pi}\left[2-1\right]=\boxed{\frac{3}{\pi}}$$

$$f(c)=\sec c\tan c=\frac{3}{\pi}$$

$$\frac{\sin c}{\cos^2 c}=\frac{3}{\pi} \qquad \sin c=\frac{3}{\pi}-\frac{3}{\pi}\sin^2 c$$

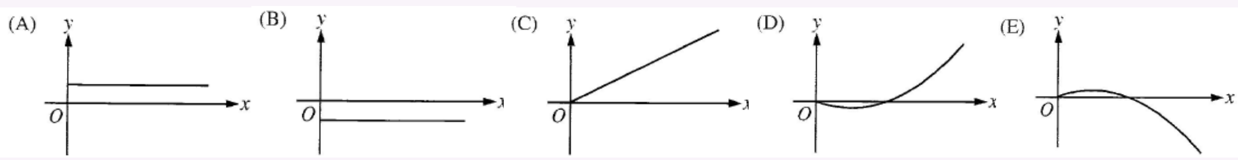
$$\frac{\sin c}{1-\sin^2 c}=\frac{3}{\pi} \qquad \frac{3}{\pi}\sin^2 c+\sin c-\frac{3}{\pi}=0$$

Closure



Graph of f

The figure above shows the graph of f. If $f(x)=\int_2^x g(t)\,dt$, which of the following could be the graph of $y=g(x)$?



Example 3

Find the average value of the function on the interval <u>without integrating</u>, by appealing to the geometry of the region between the graph and the *x*-axis.

$g(t) = 1 - \sqrt{1 - t^2}\,, \quad [-1, 1]$