

Name Answer key
Calc I H - 4.3 - 4.4 day 4

Date _____
Period _____

Setting up your own integrals

1) Find the area of the region bounded by $y = 1 + \sqrt[3]{x}$, $y = 0$, $x = 8$, and $x = 0$

$$A = \int_0^8 (1 + \sqrt[3]{x}) dx = \left(x + \frac{3}{4} x^{4/3} \right)_0^8 = 8 + \frac{3}{4} (8)^{4/3} - 0$$
$$= 8 + \frac{3}{4} (16) = \boxed{20}$$

2) Find the area of the region bounded by $y = \sqrt{x} + 2$, $y = 0$, $x = 0$, and $x = 4$

$$A = \int_0^4 (\sqrt{x} + 2) dx = \left(\frac{2}{3} x^{3/2} + 2x \right)_0^4 = \frac{2}{3} (4)^{3/2} + 2(4) - 0$$
$$= \frac{16}{3} + 8 = \frac{16}{3} + \frac{24}{3} = \frac{40}{3} = \boxed{13\frac{1}{3}}$$

3) Find the area of the region bounded by $f(x) = 3x^2 - 2x$, $y = 0$, $x = 1$, and $x = 4$

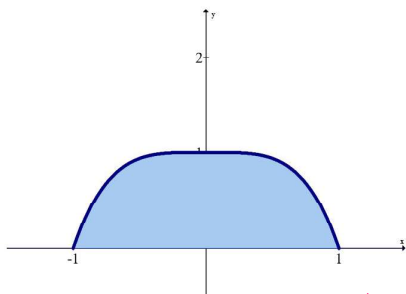
$$A = \int_1^4 (3x^2 - 2x) dx = \left(x^3 - x^2 \right)_1^4 = 4^3 - 4^2 - 1^3 + 1^2 = 48$$

4) Find the area of the region bounded by $y = \cos x$, $y = 0$, $x = \frac{\pi}{4}$, and $x = \frac{3\pi}{2}$

$$A = \int_{\pi/4}^{3\pi/2} \cos x dx = \sin x \Big|_{\pi/4}^{3\pi/2} = \sin(3\pi/2) - \sin(\pi/4)$$
$$= -1 - \frac{\sqrt{2}}{2} = \frac{-2 - \sqrt{2}}{2} \approx \boxed{-1.707}$$

Determine the area of the given region.

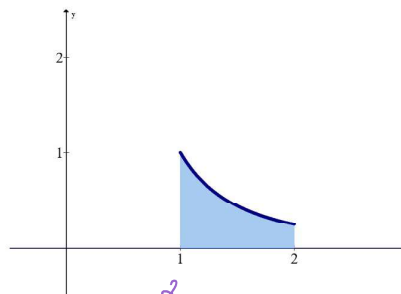
5.) $y = 1 - x^4$



$$\int_{-1}^1 (1 - x^4) dx = 2 \int_0^1 (1 - x^4) dx$$

$$= 2 \left(x - \frac{x^5}{5} \right) \Big|_0^1 = 2 \left(1 - \frac{1}{5} \right) = \boxed{\frac{8}{5}}$$

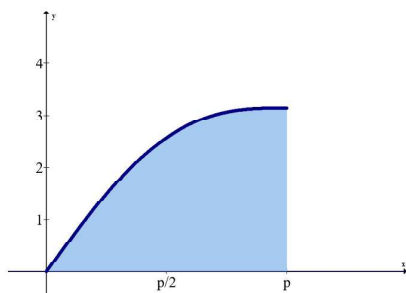
6.) $y = \frac{1}{x^2}$



$$\int_1^2 x^{-2} dx = \left. -\frac{1}{x} \right|_1^2$$

$$= -\frac{1}{2} - (-1) = \boxed{\frac{1}{2}}$$

7.) $y = x + \sin x$



$$\int_0^{\pi} (x + \sin x) dx = \left(\frac{x^2}{2} - \cos x \right) \Big|_0^{\pi}$$

$$= \frac{\pi^2}{2} - \cos(\pi) - 0 + \cos(0)$$

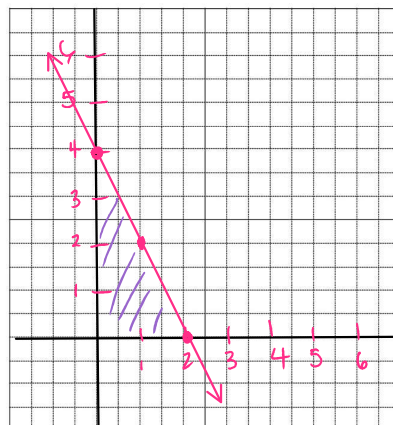
$$= \frac{\pi^2}{2} + 2 \approx 6.935$$

Sketch the region whose area is given by the definite integral. Then evaluate the integral.

8.) $\int_0^2 (4 - 2x) dx$

$$= (4x - x^2) \Big|_0^2$$

$$= 4(2) - 2^2 - 0 = \boxed{4}$$

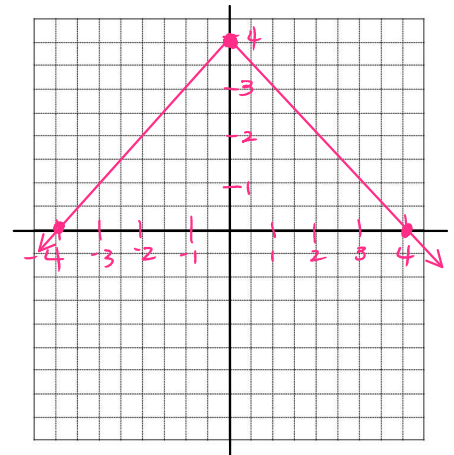


$$9.) \int_{-4}^4 (4 - |x|) dx$$

$$= \int_{-4}^0 (4+x) dx + \int_0^4 (4-x) dx$$

$$= 2 \int_0^4 (4-x) dx = 2 \left(4x - \frac{x^2}{2} \right) \Big|_0^4$$

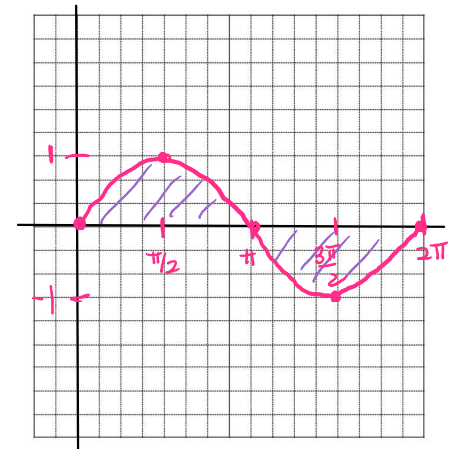
$$= 2 \left(4(4) - \frac{4^2}{2} - 0 \right) = 2(8) = \boxed{16}$$



$$10.) \int_0^{2\pi} \sin x dx$$

$$= (-\cos x) \Big|_0^{2\pi} = -\cos(2\pi) - (-\cos(0))$$

$$= -1 + 1 = \boxed{0}$$



$$11.) \int_{-1}^2 (x^2 + 2) dx$$

$$= \left(\frac{x^3}{3} + 2x \right) \Big|_{-1}^2$$

$$= \frac{8}{3} + 4 - \left(-\frac{1}{3} - 2 \right)$$

$$= \boxed{9}$$

