Do Now:
Evaluate the following integrals.

1. $\int (x+1)(3-2x-x^2)^7\,dx = \int \cancel{(x+1)}\,u^7 \frac{du}{-2\cancel{(x+1)}}$

$u = 3-2x-x^2$
$\frac{du}{dx} = -2x-2$
$du = (-2x-2)dx$
$\frac{du}{-2(x+1)} = dx$

$= -\frac{1}{2}\int u^7 du$
$= -\frac{1}{2}\left(\frac{1}{8}u^8\right) + C$
$\boxed{= -\frac{1}{16}(3-2x-x^2)^8 + C}$

2. $\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 x\,u^{1/2} \frac{du}{-2x} = \frac{1}{2}\int_0^1 x\,u^{1/2} \frac{du}{x}$

$u = 1-x^2$
$\frac{du}{dx} = -2x$
$du = -2x\,dx$
$\frac{du}{-2x} = dx$

$= \frac{1}{2}\int_0^1 u^{1/2} du$
$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^1$
$= \frac{1}{3}(1^{3/2} - 0)$
$\boxed{= \frac{1}{3}}$

Practice Problems:
Evaluate the indefinite integral.

1. $\int \frac{\cos x}{\sqrt{1-\sin x}}\,dx = \int \frac{\cos x}{\sqrt{u}} \frac{du}{-\cos x}$

$u = 1-\sin x$
$\frac{du}{dx} = -\cos x$
$du = -\cos x\,dx$
$\frac{du}{-\cos x} = dx$

$= -\int u^{-1/2} du$
$= -2u^{1/2} + C$
$\boxed{= -2\sqrt{1-\sin x} + C}$

2. $\int_0^1 2x(x^2+1)^2\,dx = \int_1^2 2x\,u^2 \frac{du}{2x}$

$u = x^2+1$
$\frac{du}{dx} = 2x$
$du = 2x\,dx$
$\frac{du}{2x} = dx$

$= \int_1^2 u^2 du$
$= \frac{1}{3}u^3\Big]_1^2$
$= \frac{1}{3}(2^3 - 1^3)$
$= \frac{1}{3}(8-1)$
$\boxed{= \frac{7}{3}}$

3. $\int (x+2)\sqrt{x-4}\,dx$

$u = x-4 \rightarrow u+4 = x$
$\frac{du}{dx} = 1$
$du = dx$

$\int (x+2)\sqrt{u}\,du$
$\int (u+4+2)\sqrt{u}\,du$
$\int u^{3/2} + 6u^{1/2} du =$
$\frac{2}{5}u^{5/2} + 4u^{3/2} + C =$
$\boxed{\frac{2}{5}(x-4)^{5/2} + 4(x-4)^{3/2} + C}$

4. $\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr = \int_5^5 \frac{5r}{u^2} \frac{du}{2r}$

$u = 4+r^2$
$\frac{du}{dr} = 2r$
$du = 2r\,dr$
$\frac{du}{2r} = dr$

$= \int_5^5 \frac{5}{2u} du$
$\boxed{= 0}$

5. $\int_0^{\pi/2} \sin x\sqrt{\cos x}\,dx = \int_1^0 \sin x\sqrt{u} \frac{du}{-\sin x}$

$u = \cos x$
$\frac{du}{dx} = -\sin x$
$du = -\sin x\,dx$
$\frac{du}{-\sin x} = dx$

$= -\int_1^0 \sqrt{u}\,du$
$= \int_0^1 \sqrt{u}\,du$
$= \frac{2}{3}u^{3/2}\Big]_0^1$

6. $\int_3^9 \frac{x+2}{\sqrt{x+6}}\,dx = \int_9^{15} \frac{u-6+2}{\sqrt{u}}\,du$

$u = x+6 \rightarrow u-6 = x$
$\frac{du}{dx} = 1$
$du = dx$

$= \int_9^{15} u^{1/2} - 4u^{-1/2} du$
$= \frac{2}{3}u^{3/2} - 8u^{1/2}\Big]_9^{15}$

$= \frac{2}{3}[1-0]$

$\boxed{= \frac{2}{3}}$

$= \frac{2}{3}(15)^{3/2} - 8(15)^{1/2} - \frac{2}{3}(9)^{3/2} + 8(9)^{1/2}$

$= \frac{2}{3}(15\sqrt{15}) - 8\sqrt{15} - 18 + 24$

$= 10\sqrt{15} - 8\sqrt{15} + 6$

$\boxed{= 2\sqrt{15} + 6}$

7. $\int \tan^2 x \sec^2 x\,dx = \int u^2 \sec^2 x \frac{du}{\sec^2 x}$

$u = \tan(x)$

$\frac{du}{dx} = \sec^2(x)$

$du = \sec^2(x)\,dx$

$\frac{du}{\sec^2(x)} = dx$

$= \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$\boxed{= \frac{1}{3}\tan^3(x) + C}$

8. $\int 4(6x-1)^{2/3}\,dx = 4\int u^{2/3} \frac{du}{6}$

$u = 6x-1$

$\frac{du}{dx} = 6$

$du = 6dx$

$dx = \frac{du}{6}$

$= \frac{2}{3}\int u^{\frac{2}{3}}\,du$

$= \frac{2}{3}\left(\frac{3}{5}u^{5/3}\right) + C$

$\boxed{= \frac{2}{5}(6x-1)^{5/3} + C}$

9. $\int_0^{\pi/4} \sin 2x\,dx = \int_0^{\frac{\pi}{2}} \sin(u)\frac{du}{2}$

$u = 2x$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin(u)\,du$

$= \frac{-1}{2}[\cos u]_0^{\frac{\pi}{2}}$

$= \frac{-1}{2}\left(\cos(\frac{\pi}{2}) - \cos(0)\right)$

$= \frac{-1}{2}(0-1)$

$\boxed{= \frac{1}{2}}$

10. $\int (x+2)\sqrt{x^2+4x-3}\,dx =$

$u = x^2+4x-3$

$\frac{du}{dx} = 2x+4$

$du = 2x+4\,dx$

$\frac{du}{2(x+2)} = dx$

$\int \cancel{(x+2)}\sqrt{u}\frac{du}{2\cancel{(x+2)}} =$

$\frac{1}{2}\int \sqrt{u}\,du =$

$\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C =$

$\boxed{\frac{1}{3}(x^2+4x-3)^{3/2} + C}$

11. $\int \frac{x}{(x+3)^2}\,dx = \int \frac{x}{u^2}\,du$

$u = x+3$

$\frac{du}{dx} = 1$

$du = dx$

$u-3 = x$

$= \int \frac{u-3}{u^2}\,du$

$= \int u^{-1} - 3u^{-2}\,du$

$= \ln|u| + 3u^{-1} + C$

$\boxed{= \ln|x+3| + \frac{3}{x+3} + C}$

12. $\int \frac{x^3+2}{x^4+8x}\,dx = \int \frac{x^3+2}{u}\frac{du}{4(x^3+2)}$

$u = x^4 + 8x$

$\frac{du}{dx} = 4x^3 + 8$

$du = 4x^3 + 8dx$

$\frac{du}{4(x^3+2)} = dx$

$= \frac{1}{4}\int \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u| + C$

$\boxed{= \frac{1}{4}\ln|x^4+8x| + C}$